V8 - Sampling Distribution Theory Part 3

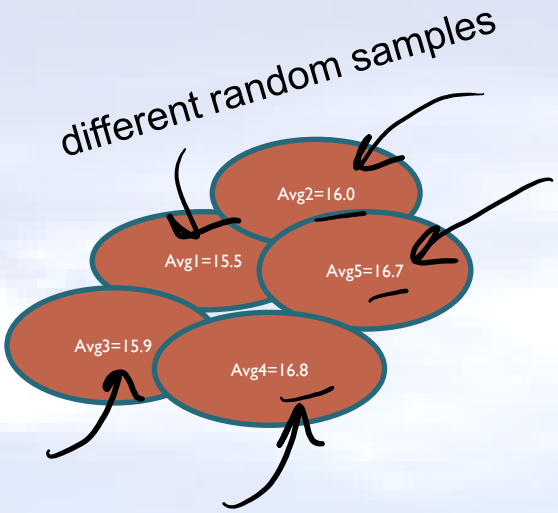
Course: Statistical Testing & Regression
Dr. Renee Clark
Swanson School of Engineering
Industrial Engineering
University of Pittsburgh

Sampling Distribution Theory – Part 3

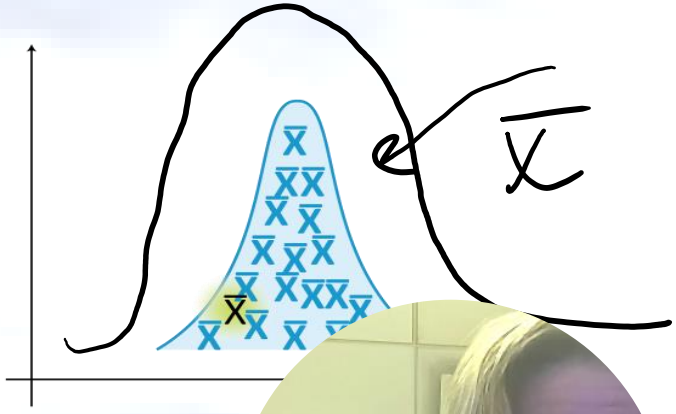
- REVIEW: what is a sampling distribution?
- Shape of sampling distribution of mean when population is normal

What is a sampling distribution?

Distribution of a statistic such as the sample average



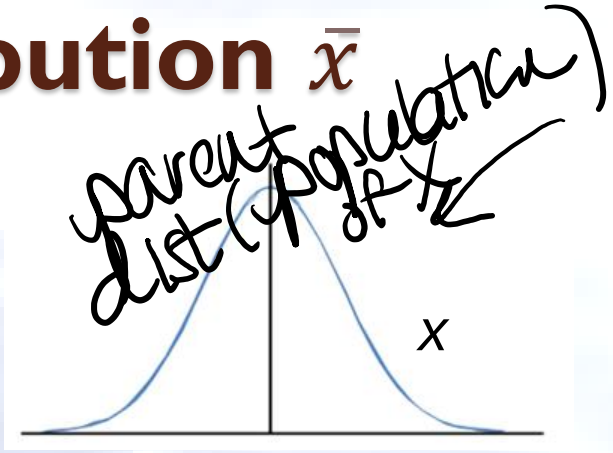
Plot these averages; form distribution



Shape of Sampling Distribution $\bar{x}$

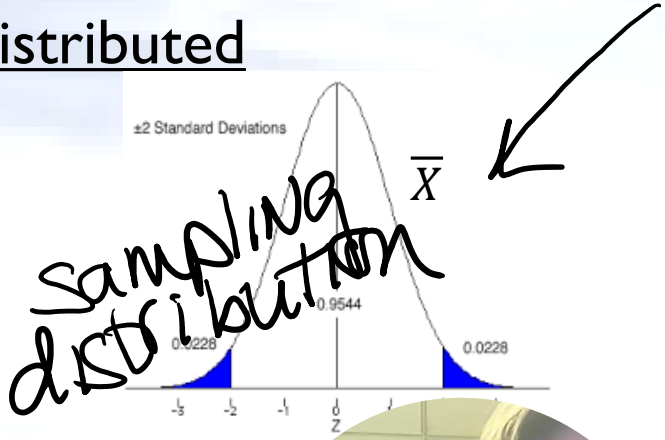
If population (parent) is normally distributed

If $x \sim N$



Then sampling distribution will be normally distributed

$\bar{x} \sim N$



Why?

If $X_1, X_2, \ldots, X_n$ are independent random variables having normal distributions with means $\mu_1, \mu_2, \ldots, \mu_n$ and variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$, respectively, then the random variable

$$Y = a_1X_1 + a_2X_2 + \cdots + a_nX_n$$

has a normal distribution with mean

$$\mu_Y = a_1\mu_1 + a_2\mu_2 + \cdots + a_n\mu_n$$

and variance

$$\sigma_Y^2 = a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + \cdots + a_n^2\sigma_n^2.$$

$X \sim N$ $X_i \sim N$

linear combination of X_i's

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

$$\bar{X} = \frac{1}{n}X_1 + \frac{1}{n}X_2 + \cdots + \frac{1}{n}X_n$$

$\bar{X}$ is therefore a linear combination of the X_i's with constant 1/n for coefficient of each term

Acknowledgement

This material is based upon work partially supported by the National Science Foundation under Grant# 2335802. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

Under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon this work, you may distribute the resulting work only under the same or similar license to this one.

THE END