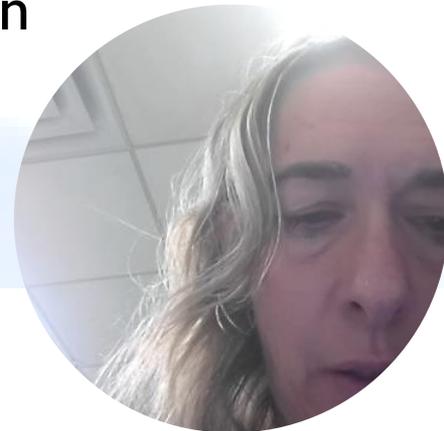


V39 - Multiple Linear Regression - Part I

- Introduction to Multiple Linear Regression
- Multicollinearity

Course: Statistical Testing & Regression
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Regression Analysis

- Simple Linear Regression
 - 1 independent variable
 - Need a foundation!
- Reasonable at times
- Many scientific problems or real-world applications tend to be more complex
 - Require more than 1 independent variable to explain variation & make good predictions
 - multiple regression model
 - > 1 independent variable



Multiple Linear Regression Model

$y = \beta_0 + \beta_1 x + \varepsilon$ simple linear regression
SLR

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots + \beta_k x_k + \varepsilon$ MLR
"k" indep variables

$\rightarrow \varepsilon \sim N(0, \sigma^2)$

Each β_i estimated via Least Squares Method \rightarrow obtain b_i 's

$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + \dots + b_k x_k$



Multicollinearity

- ❖ **Multicollinearity** refers to presence of linear relationships between *independent variables*

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \dots + \beta_k x_k + \varepsilon$$

- ❖ Investigate **multicollinearity** via correlation coefficient (r)



Multicollinearity

Correlation matrix for 4 independent variables: x1, x2, x3, x4

Correlation: x1, x2, x3, x4

	x1	x2	x3
x1	1.000	0.952	0.534
x2	0.952	1.000	0.263
x3	0.534	0.263	1.000
x4	0.390	0.155	0.784

Cell Contents: Pearson correlation
P-Value

Handwritten notes on the table:
 - A star is next to the x1 header.
 - A red box highlights the x1-x2 correlation (0.952) with a red arrow pointing to 'r'.
 - A yellow circle highlights the x2-x3 correlation (0.263).
 - A purple box highlights the x3-x4 correlation (0.784) with a purple arrow pointing to 'P'.
 - A purple circle highlights the x4 header.

★ Substantial correlations ($p < 0.05$):

x1 & x2
x3 & x4

$H_0: \rho = \phi$ X
 $H_1: \rho \neq \phi$ ✓





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THE END

