

V38 - Simple Linear Regression – Part 3

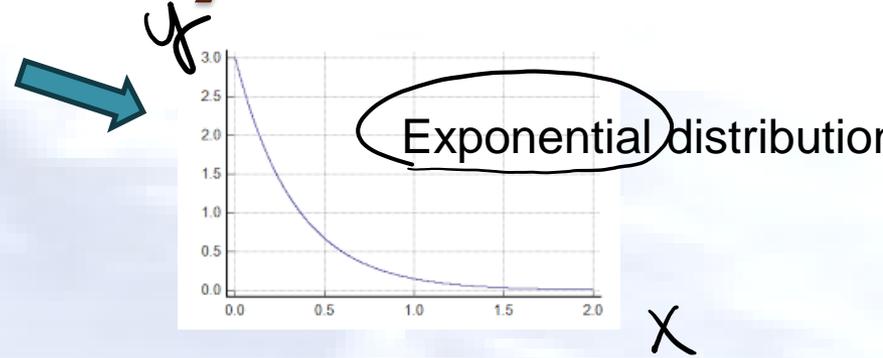
- Transformations to linearize a relationship between x & y

Course: Statistical Testing & Regression
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What if x & y not linearly related?

- ❖ As evident via scatter plot
- ❖ Or via theory/experience



- ❖ Can we run a linear regression?

- ✓ Possibly...

- May be able to transform x and/or y to create linear relationship.

- ❖ If create linear model with curved data, poor fit



What if x & y not linearly related?

- Transformation re-expresses x & y

- x* and y*

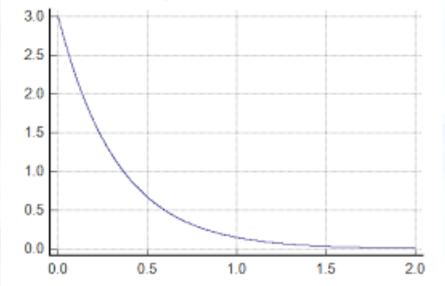
❖ Example of transformation:

ln(y) = y* ✓

= ln(y)

- Run regression of y* versus x

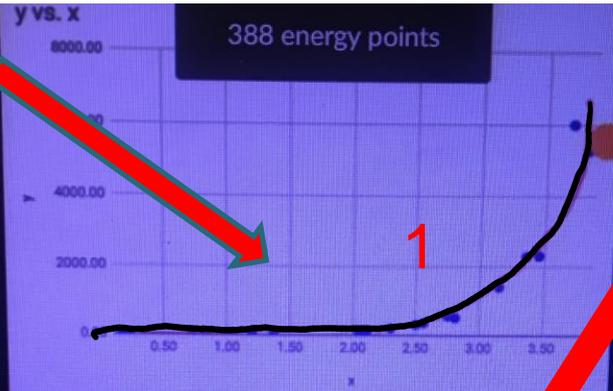
Exponential distribution



Transformations

1

x	y
3.46	2307.23
3.36	2290.87
1.37	23.44
3.82	5285.55
2.75	562.34
0.73	6.44
0.17	1.77
1.20	19.02
2.30	199.53
2.06	114.82
2.56	363.08
2.04	131.58
3.70	6014.25
2.50	316.23
1.02	12.57
2.11	128.82
3.15	1412.54
0.83	5.41
2.81	516.52
0.22	1.70



log(y)

2

x



Transformations

❖ Common transformations for linearity:

\log \rightarrow \ln

➤ Logarithm (base 10 or base e)

➤ Reciprocal

➤ x

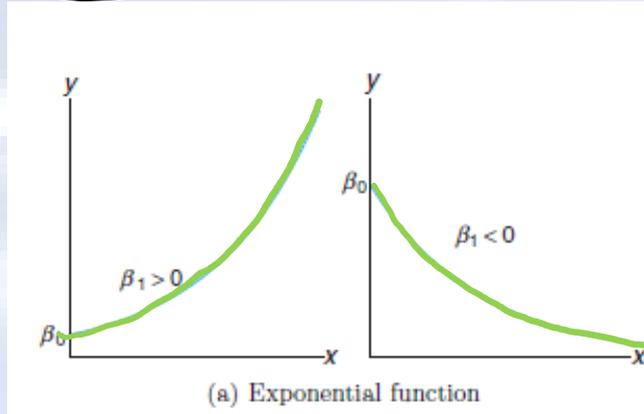
➤ y

➤ or both



Example: Exponential Relationship

Assume scatterplot looks like:



Suspect x and y **exponentially** related

$$y = \beta_0 e^{\beta_1 x}$$

Appropriate Transformation:

- $\ln y = y^*$ *ln y*
- Regress y^* on x
- Parameters estimated via Least Squares



Common Transformations to Linearize x & y

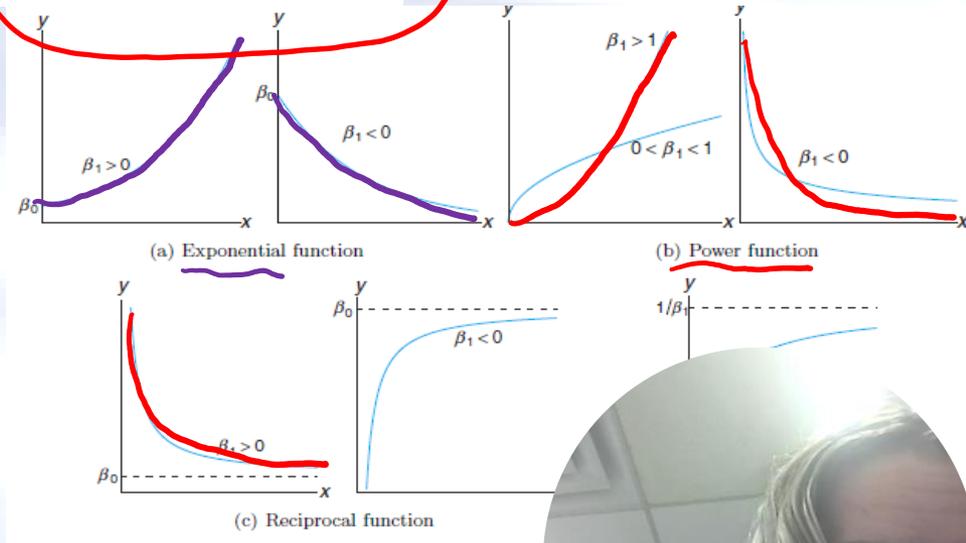
Table 11.6: Some Useful Transformations to Linearize

Functional Form Relating y to x	Proper Transformation	Form of Simple Linear Regression
Exponential: $y = \beta_0 e^{\beta_1 x}$	$y^* = \ln y$ *	Regress y^* against x
Power: $y = \beta_0 x^{\beta_1}$ ✓	$y^* = \log y$; $x^* = \log x$ *	Regress y^* against x^*
Reciprocal: $y = \beta_0 + \beta_1 \left(\frac{1}{x}\right)$	$x^* = \frac{1}{x}$ *	Regress y against x^*
Hyperbolic: $y = \frac{x}{\beta_0 + \beta_1 x}$	$y^* = \frac{1}{y}$; $x^* = \frac{1}{x}$ *	Regress y^* against x^*

❖ Trial & error process

- May not know true relationship between x & y
- May be hard to differentiate among relationships
 - Exponential vs. power

❖ Typical to try various transformations, plot transformed data, and choose one that appears most linear.





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THE END

