V38 - Simple Linear Regression – Part 3

- Transformations to linearize a relationship between x & y

Course: Statistical Testing & Regression
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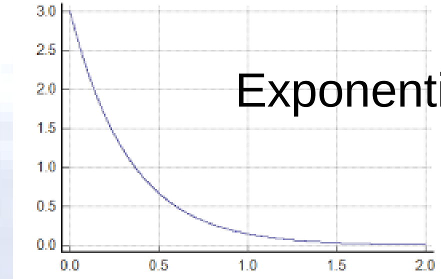
What if x & y not linearly related?



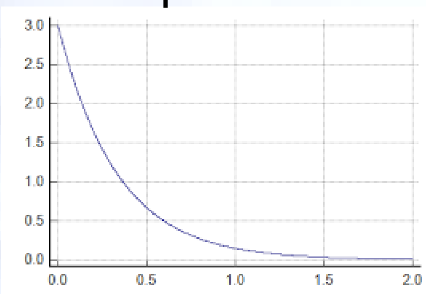
- As evident via ___________
- Or via theory/experience

- Can we run a linear regression?
  - Possibly...
  - May be able to __________ **x** and/or **y** to create linear relationship.
- If create linear model with curved data, poor ___

What if x & y not linearly related?

- **Transformation** *re-expresses* x & y
  - ____ and ____

Exponential distribut

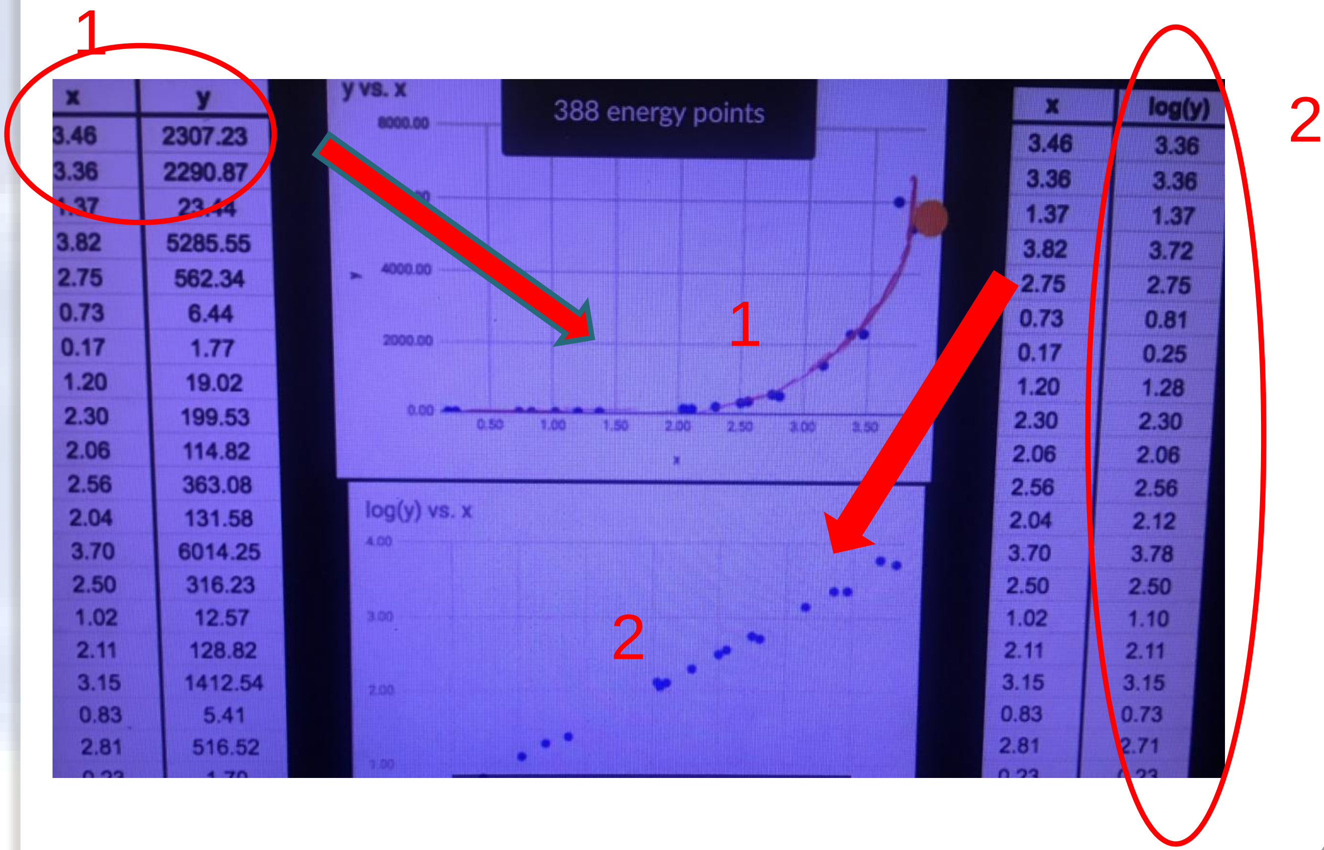


❖ Example of transformation:

ln(y) = y*

- Run regression of y* versus x

Transformations

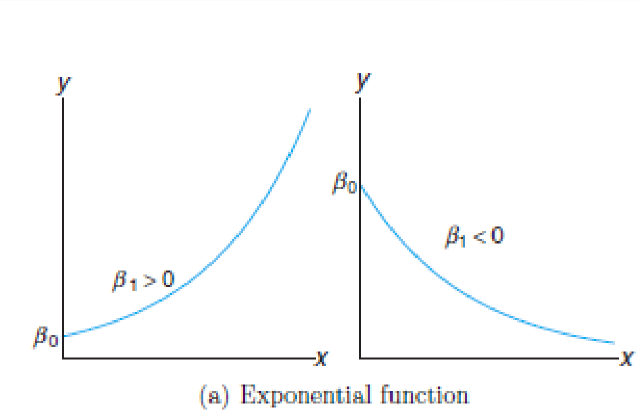


Transformations

- Common transformations for linearity:
  - **Logarithm** (base 10 or base e)
  - **Reciprocal**
  - x
  - y
  - or both

Example: Exponential Relationship

Assume scatterplot looks like:



Suspect x and y **exponentially** related

$$y = \boldsymbol{\beta}_0 \, e^{\boldsymbol{\beta}_1 x}$$

Appropriate Transformation:

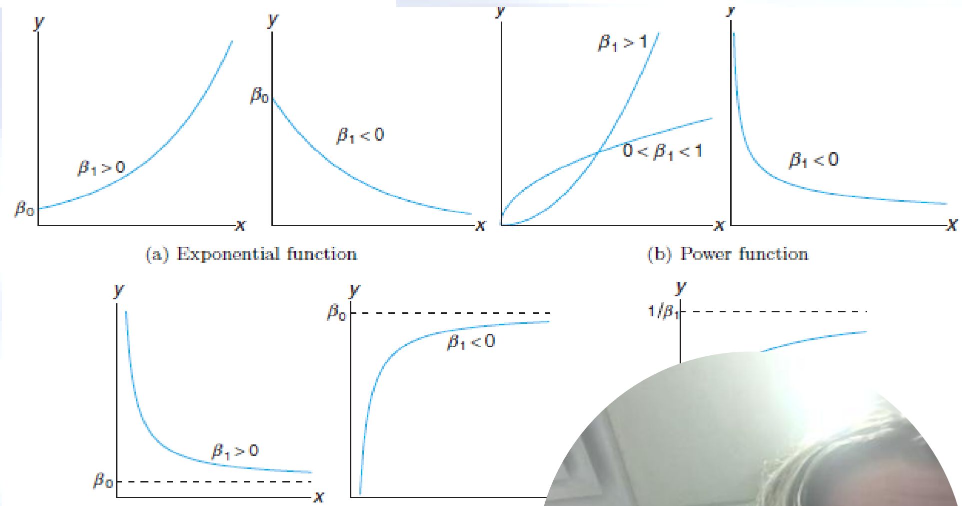
- ln y = y*
- Regress y* on x
- Parameters estimated via Least Squares

Common Transformations to Linearize x & y

Table 11.6: Some Useful Transformations to Linearize

Functional Form Relating y to x	Proper Transformation	Form of Simple Linear Regression
Exponential: $y = \beta_0 e^{\beta_1 x}$	$y^* = \ln y$	Regress y^* against x
Power: $y = \beta_0 x^{\beta_1}$	$y^* = \log y; \quad x^* = \log x$	Regress y^* against x^*
Reciprocal: $y = \beta_0 + \beta_1\left(\frac{1}{x}\right)$	$x^* = \frac{1}{x}$	Regress y against x^*
Hyperbolic: $y = \frac{x}{\beta_0 + \beta_1 x}$	$y^* = \frac{1}{y}; \quad x^* = \frac{1}{x}$	Regress y^* against x^*

- Trial & error process
  - May not know true relationship between ____ & ____
  - May be hard to differentiate among relationships
    - Exponential vs. power
- Typical to try various transformations, plot transformed data, and choose one that appears most ____________.



(a) Exponential function (b) Power function (c) Reciprocal function

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