

V38 - Simple Linear Regression – Part 3

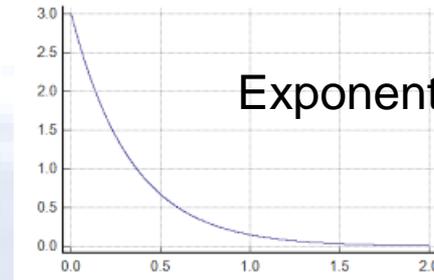
- Transformations to linearize a relationship between x & y

Course: Statistical Testing & Regression
Dr. Renee Clark
Swanson School of Engineering
Industrial Engineering
University of Pittsburgh



What if x & y not linearly related?

- ❖ As evident via _____
- ❖ Or via theory/experience



- ❖ Can we run a linear regression?
 - Possibly...
 - May be able to _____ x and/or y to create linear relationship.
- ❖ If create linear model with curved data, poor _____



What if x & y not linearly related?

- Transformation *re-expresses* x & y

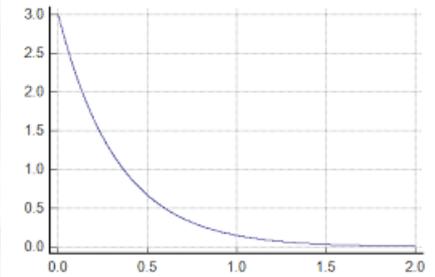
- _____ and _____

- Example of transformation:

$$\ln(y) = y^*$$

- Run regression of y^* versus x

Exponential distribution



Transformations



Transformations

❖ Common transformations for linearity:

➤ Logarithm (base 10 or base e)

➤ Reciprocal

➤ x

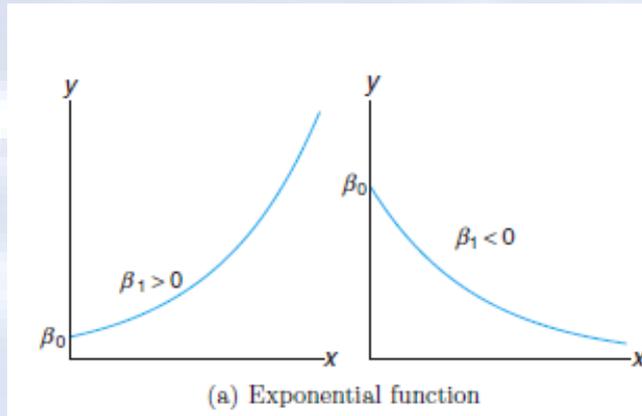
➤ y

➤ or both



Example: Exponential Relationship

Assume scatterplot looks like:



Suspect x and y **exponentially** related

$$y = \beta_0 e^{\beta_1 x}$$

Appropriate Transformation:

- $\ln y = y^*$
- Regress y^* on x
- Parameters estimated via Least Squares



Common Transformations to Linearize x & y

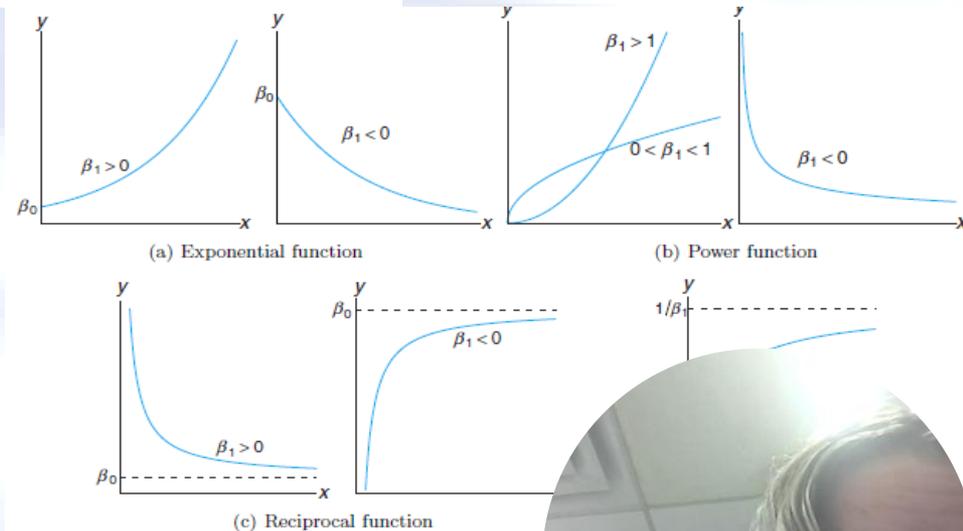
Table 11.6: Some Useful Transformations to Linearize

Functional Form Relating y to x	Proper Transformation	Form of Simple Linear Regression
Exponential: $y = \beta_0 e^{\beta_1 x}$	$y^* = \ln y$	Regress y^* against x
Power: $y = \beta_0 x^{\beta_1}$	$y^* = \log y; \quad x^* = \log x$	Regress y^* against x^*
Reciprocal: $y = \beta_0 + \beta_1 \left(\frac{1}{x}\right)$	$x^* = \frac{1}{x}$	Regress y against x^*
Hyperbolic: $y = \frac{x}{\beta_0 + \beta_1 x}$	$y^* = \frac{1}{y}; \quad x^* = \frac{1}{x}$	Regress y^* against x^*

❖ Trial & error process

- May not know true relationship between ____ & ____
- May be hard to differentiate among relationships
 - Exponential vs. power

❖ Typical to try various transformations, plot transformed data, and choose one that appears most _____.





Acknowledgement

This material is based upon work partially supported by the National Science Foundation under Grant# 2335802. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.





You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

Under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon the work, you may distribute the resulting work only under the same or similar license to this one.





THE END

