V32 - Hypothesis Testing – Part 8

- Chi-squared test statistic involving population variance
- Visual review of chi-squared distribution
- F test statistic involving ratio of 2 population variances (independent populations)
- Visual review of F distribution

Course: Statistical Testing & Regression
Dr. Renee Clark
Swanson School of Engineering
Industrial Engineering
University of Pittsburgh

Inference for One Variance σ^2

Normally Distributed Population σ^2 unknown

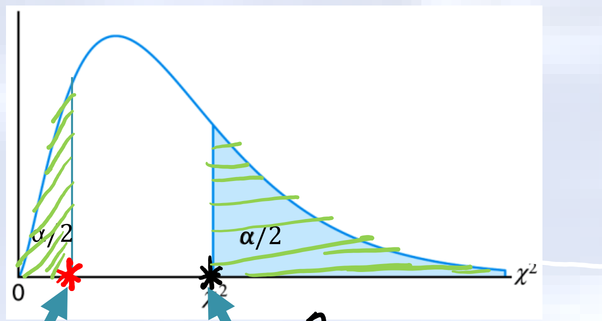
Sample

- Point estimate of $\boldsymbol{\sigma^2}$ is s^2

χ^2

- Statistic $\frac{(n-1)s^2}{\sigma^2} = \chi^2$ has a chi-squared distribution with $n-1$ degrees of freedom
- Population must be normally distributed

Review – Chi Squared Distribution



$\chi^2_{1-\frac{\alpha}{2}}$

$\chi^2_{\alpha/2}$

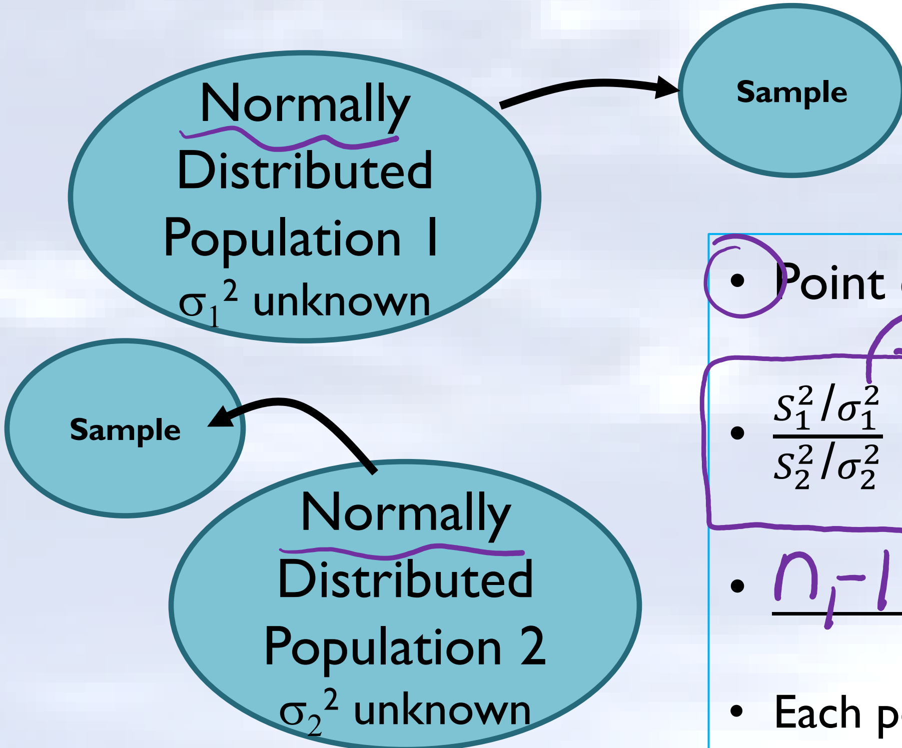
Inference for One Variance σ^2

$H_0: \sigma^2 = \sigma_0^2$

$H_1: \sigma^2 \neq \sigma_0^2$

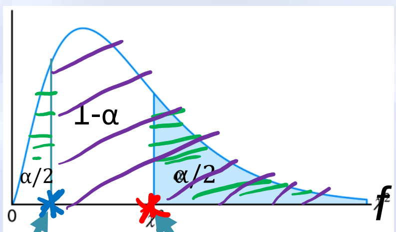
Test Statistic: $X^2 = \dfrac{(n-1)S^2}{\sigma_0^{\ 2}}$

Inference for Ratio of Two Variances σ_1^2/σ_2^2



- Point estimate for $\boldsymbol{\sigma_1^2/\sigma_2^2}$ is $\frac{s_1^2}{s_2^2}$
- $\boxed{\dfrac{s_1^2/\sigma_1^2}{s_2^2/\sigma_2^2}} = \dfrac{\sigma_2^2 s_1^2}{\sigma_1^2 s_2^2}$ has an F distribution
- n_1-1 and n_2-1 degrees of freedom
- Each population must be normally distributed

Review – F Distribution



$f_{1-\alpha/2}$

$f_{\alpha/2}$

Inference for Ratio of Two Variances σ_1^2/σ_2^2

$$\boxed{H_0 : \sigma_1^2 = \sigma_2^2}$$

$$H_0 : \frac{\sigma_1^2}{\sigma_2^2} = 1$$

$$H_0 : \frac{\sigma_2^2}{\sigma_1^2} = 1$$

Test statistic

$$F = \frac{S_1^2/\cancel{\sigma_1^2}}{S_2^2/\cancel{\sigma_2^2}} = \frac{\cancel{\sigma_2^2} S_1^2}{\cancel{\sigma_1^2} S_2^2}$$

$$F = \frac{S_1^2}{S_2^2}$$

$\rightarrow$ $v_1 = n_1 - 1$ and $v_2 = n_2 - 1$

Acknowledgement

This material is based upon work partially supported by the National Science Foundation under Grant# 2335802. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

THE END