V29 - Hypothesis Testing – Part 5

- Paired data (dependent populations)
- Use of T and Z distributions for inference on difference in two means from dependent populations

Course: Statistical Testing & Regression
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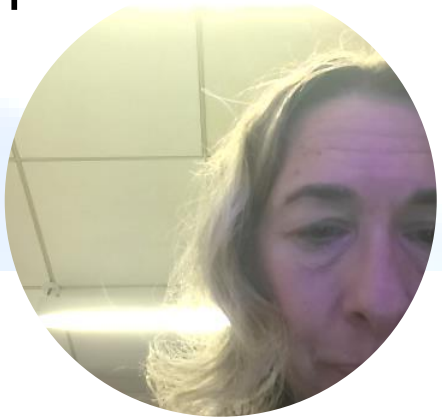


Paired (Dependent) Data Setups Occur When...

- Same subject - 2 different conditions
  - Before and after study (e.g., testing fitness program or teaching method)
  - Brand A vs. Brand B wear (Left & right foot of same person)
- "Matched" subjects (different persons)
  - Brother & sister pair
  - Husband & wife pair

- Reduces unwanted variability inherent with independent groups

Inference on difference in two means $\mu_1 - \mu_2$
(dependent populations or paired data)
σ^2_d unknown & small n

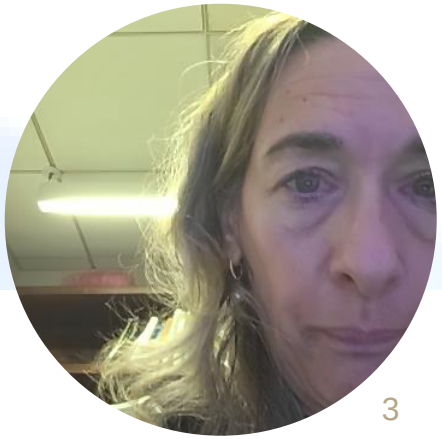
$\mu_D = \phi$

$H_0: \mu_1 - \mu_2 = \mu_D = 0 = d_0$

Test statistic: $T = \dfrac{\bar{d} - d_0}{s_d/\sqrt{n}}$

i	X1 (Before)	X2 (After)	d_i	
1	1	2	1	2-1
2	3	5	2	5-3
3	4	7	3	
4	5	9	4	
5	6	11	5	
6	7	13	6	
			3.5	AVG = $\bar{d}$
			1.9	STDEV = s_d

- $\bar{d}$ is mean of sample differences d_i's
- S_d is standard deviation of d_i's
- n is number of pairs
- d_i's ~ N



Inference on difference in two means $\mu_1 - \mu_2$
(dependent populations or paired data)

σ^2_d known

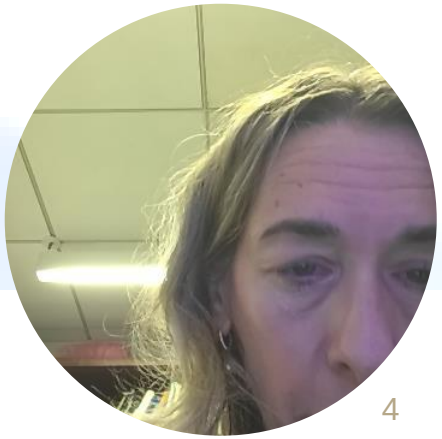
$H_0: \mu_1 - \mu_2 = \mu_D = 0 = d_0$

Test statistic: $Z = \dfrac{\bar{d} - d_0}{\sigma_d / \sqrt{n}}$ *

$\bar{d} \sim N$

(1) n large $\bar{d} \sim N$ by CLT

(2) $d_i \sim N \Rightarrow \bar{d} \sim N$



Inference on difference in two means $\mu_1 - \mu_2$
(dependent populations or paired data)
σ^2_d unknown & large n

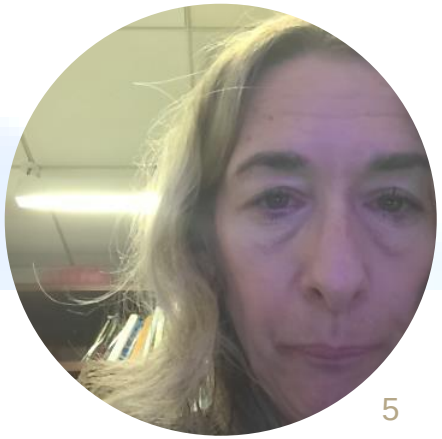
$H_0: \mu_1 - \mu_2 = \mu_D = 0 = d_0$

Test statistic: $Z \sim \dfrac{\bar{d} - d_0}{s_d/\sqrt{n}}$

σ_d s_d

S_d good estimator of σ_d since n large ($n \geq 30$)

$\bar{d} \sim N$ by CLT ($n \geq 30$)



Inference on difference in two means $\mu_1 - \mu_2$

Relationship to Confidence Interval

$H_0: \mu_1 = \mu_2$ $\quad d_0 = \mu_1 - \mu_2 = \mu_D = \phi$

(0.4 , 1.5) ϕ is not plausible for $\mu_D = \mu_1 - \mu_2$

Reject ϕ as plausible value for μ_D

(-0.95 , 1.65)

neg pos

ϕ is plausible value for $\mu_1 - \mu_2 = \mu_D$

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