V28 - Hypotheses Testing – Part 4

- Use of T distribution for inference on difference in two means from independent populations

Course: Statistical Testing & Regression
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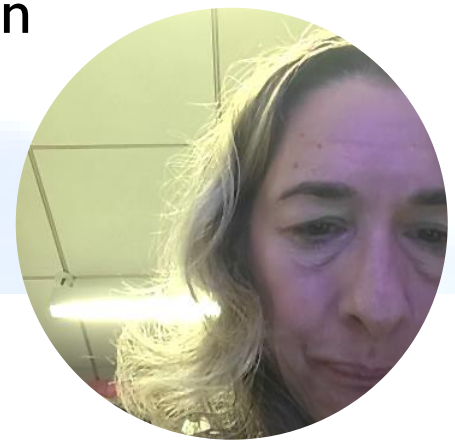

Inference on difference in two means $\mu_1 - \mu_2$ (independent populations)

- σ_1^2 and σ_2^2 unknown and n_1 and n_2 small
- We use T distribution

Unknown Variances but believed $\sigma_1^2 = \sigma_2^2$

- When population variances believed equal, use pooled estimate of variance

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Inference on difference in two means $\mu_1 - \mu_2$ (independent populations)

Case of σ_1^2 and σ_2^2 unknown and small samples n_1 & n_2

$\sigma_1^2 = \sigma_2^2$

$H_0: \mu_1 = \mu_2$ ($d_0 = \mu_1 - \mu_2 = 0$)

Test statistic $T = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_0}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

$(\mu_1 - \mu_2)$

$X_1 \sim$ N and $X_2 \sim$ N

Use pooled estimate of variance s_p^2

degrees of freedom $v = n_1 + n_2 - 2$

Inference on difference in two means $\mu_1 - \mu_2$ (independent populations)

Case of σ_1^2 and σ_2^2 unknown and small samples n_1 & n_2

$\sigma_1^2 \neq \sigma_2^2$

Cannot use pooled estimate of variance

$H_0: \mu_1 = \mu_2$ ($d_0 = \mu_1 - \mu_2 = 0$)

Test statistic $T = \dfrac{(\bar{X}_1 - \bar{X}_2) - d_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$X_1 \sim$ N and $X_2 \sim$ N

degrees of freedom

$$v = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{[(s_1^2/n_1)^2/(n_1-1)] + [(s_2^2/n_2)^2/(n_2-1)]}$$

9.54

df = 9

Round down to largest whole number if has a decimal component.

Inference on difference in two means $\mu_1 - \mu_2$

Relationship to Confidence Interval

$H_0: \mu_1 = \mu_2$

$d_0 = \mu_1 - \mu_2 = \phi$

T, 2

(2.95, 3.65)

(-1.25, 2.25)

LL UL

ϕ is not plausible

Reject ϕ as plausible for $\mu_1 - \mu_2$

ϕ is a plausible value for $\mu_1 - \mu_2$

Fail to Reject ϕ as plausible for $\mu_1 - \mu_2$

LL < $\mu_1 - \mu_2$ < UL

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