V27 - Hypothesis Testing - Part 3

- Use of Z distribution for inference on difference in two means from independent populations

Course: Statistical Testing & Regression
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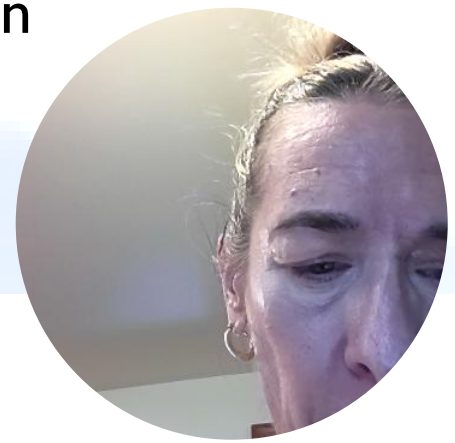

Inference on difference in two means $\mu_1 - \mu_2$
(independent populations)
Case of σ_1^2 and σ_2^2 known

Test Statistic $Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

$$E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$$

$$\text{Var}(\bar{X}_1 - \bar{X}_2) = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

$\bar{x}_1 - \bar{x}_2$ must be _____

- n_1 and n_2 each ≥ _____ ( ) OR x_1 and x_2 each ~ →
- $\bar{x}_1 - \bar{x}_2$ will be normally distributed (linear combination)

Inference on difference in two means $\mu_1 - \mu_2$ (independent populations)

Case of σ_1^2 and σ_2^2 known

$H_0: \mu_1 = \mu_2$ $\quad$ $H_0: \mu_1 - \mu_2 = \_\_\_ = d_0$ (typically $d_0 = 0$ but can be non-zero)

$H_1: \mu_1 \neq \mu_2$

Proof by contradiction: insert $d_0 = 0$

Test statistic Z = $$\frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

Inference on difference in two means $\mu_1 - \mu_2$ (independent populations)

Case of σ_1^2 and σ_2^2 unknown (but n_1 & n_2 large)

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Test statistic $$Z \sim \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$\text{Var}\,(\bar{X}_1 - \bar{X}_2) = \frac{{\sigma_1}^2}{n_1} + \frac{{\sigma_2}^2}{n_2}$$

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