V26 - Hypothesis Testing - Part 2

- Cases involving inference for one mean μ
  - Z distribution vs.
  - T distribution
- Proof by Contradiction
- Relationship of confidence interval to hypothesis test

Course: Statistical Testing & Regression
Dr. Renee Clark
Swanson School of Engineering
Industrial Engineering
University of Pittsburgh

Inference for Mean μ

Case of σ^2 Known

$H_0: \mu = \mu_0$ hypothesized value

$H_1: \mu \neq \mu_0$ new claim or belief

Trying to show or PROVE

Test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Insert hypothesized value μ_0 for μ

proof by contradiction

?

$\bar{X}$ must be normally distributed

1) $X \sim N \Rightarrow \bar{X} \sim N$

OR

2) n large (≥ 30) $\Rightarrow \bar{X} \sim N$ by CLT

Relationship of HT to Confidence Interval

One-Sample Z

Descriptive Statistics

N	Mean	SE Mean	95% CI for μ
49	51.300	0.571	(50.180, 52.420)

μ: mean of Sample
Known standard deviation = 4

Test

Null hypothesis $H_0: \mu = 50$
Alternative hypothesis $H_1: \mu \neq 50$

Z-Value	P-Value
2.27	0.023

$\mu_0 = 50$
is outside *the CI*

for μ

Not a plausible value at this α
Reject this value for μ *!*

For 2-sided HT, since μ_o outside the $100(1-\alpha)\%$ confidence interval, Reject H_o at α.

CI's and HT's are equivalent forms of inference!

$100(1-\alpha)\%$ confidence interval for μ

$H_0\colon \mu = \mu_0$
$H_1\colon \mu \neq \mu_0$

at α

Reject Ho at α if μ_o outside CI

Inference for Mean μ

Case of σ^2 unknown & n small

What if population variance σ^2 unknown & *n* small?

use T distribution

Inference for Mean μ

Case of σ^2 unknown & n small

Test Statistic:

insert μ_0 for μ

proof by contradiction

$$T = \frac{\overline{X} - \mu_0}{S / \sqrt{n}}$$

sample std dev.

- $n - 1$ degrees of freedom
- $X_1, \ldots, X_n$ from *normal* population

$X \sim N$

Inference for Mean μ

Case of σ^2 unknown & n large

What if population variance σ^2 unknown & *n* large?

use Z distribution *since* n large

$s \approx \sigma$

$s^2 \approx \sigma^2$

Inference for Mean μ

Case of σ^2 unknown & n large

$\boxed{s \approx \sigma}$

Test Statistic:

$$Z = \frac{\bar{X} - \mu_0}{\cancel{\sigma} / \sqrt{n}}$$

s

insert μ_0 for μ

proof by contradiction

n large, $\bar{X} \sim N$ by CLT

Benefit: CLT applies with Z!

Don't have to assume population of X is normal (T).

Acknowledgement

This material is based upon work partially supported by the National Science Foundation under Grant# 2335802. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

Under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upo work, you may distribute the resulting work only u same or similar license to this one.

THE END