V26 - Hypothesis Testing - Part 2

- Cases involving inference for one mean μ
  - Z distribution vs.
  - T distribution
- Proof by Contradiction
- Relationship of confidence interval to hypothesis test

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Inference for Mean μ

Case of σ^2 Known

H_o: $\mu = \mu_0$

H_1: $\mu \neq \mu_0$

Test statistic

$$Z = \frac{\bar{X} - \mu_0}{\sigma / \sqrt{n}}$$

Insert hypothesized value _____ for

_____

$\bar{X}$ must be normally distributed

1) X ~

OR

2) n large (_____)

Relationship of HT to Confidence Interval

One-Sample Z

Descriptive Statistics

N	Mean	SE Mean	95% CI for μ
49	51.300	0.571	(50.180, 52.420)

μ: mean of Sample
Known standard deviation = 4

Test

Null hypothesis $H_0: \mu = 50$
Alternative hypothesis $H_1: \mu \neq 50$

Z-Value	P-Value
2.27	0.023

$\mu_0 = 50$
is _____ the CI

Not a plausible value at this α
Reject this value for ____ !

For 2-sided HT, since μ_o outside the 100(1- α)% confidence interval, Reject H_o at α.

CI's and HT's are equivalent forms of inference!

$100(1 - \alpha)\%$ confidence interval for μ

$H_0: \mu = \mu_0$
$H_1: \mu \neq \mu_0$

at α

Reject Ho at α if μ_o outside CI

Inference for Mean μ

Case of σ^2 unknown & n small

What if population variance σ^2 unknown & *n* small?

_____

Inference for Mean μ

Case of σ^2 unknown & n small

Test Statistic:

insert _____ for

_____

$$T = \frac{\overline{X} - \mu_0}{S / \sqrt{n}}$$

- $n - 1$ degrees of freedom
- $X_1, \ldots, X_n$ from *normal* population

Inference for Mean μ

Case of σ^2 unknown & n large

What if population variance σ^2 unknown & *n* large?

_____ *since*

_____

Inference for Mean μ

Case of σ^2 unknown & n large

Test Statistic:

insert _____ for

$$Z = \frac{\bar{X} - \mu_0}{\cancel{\sigma} / \sqrt{n}}$$

s

_____

Benefit: CLT applies with Z!

Don't have to assume population of X is normal (T).

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