V21 - Estimation - Part 4

❑ Topics relevant to estimating difference in independent means

   ❑ Standardization of the difference to Z

   ❑ Normality of difference

   ❑ Pooled estimate of the sample variance

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Transformation of difference in 2 sample means to Z

$$\left(\overline{X}_1 - \overline{X}_2 \right)$$

Population 1
σ_1^2 **known**

Sample n_1

Population 2
σ_2^2 **known**
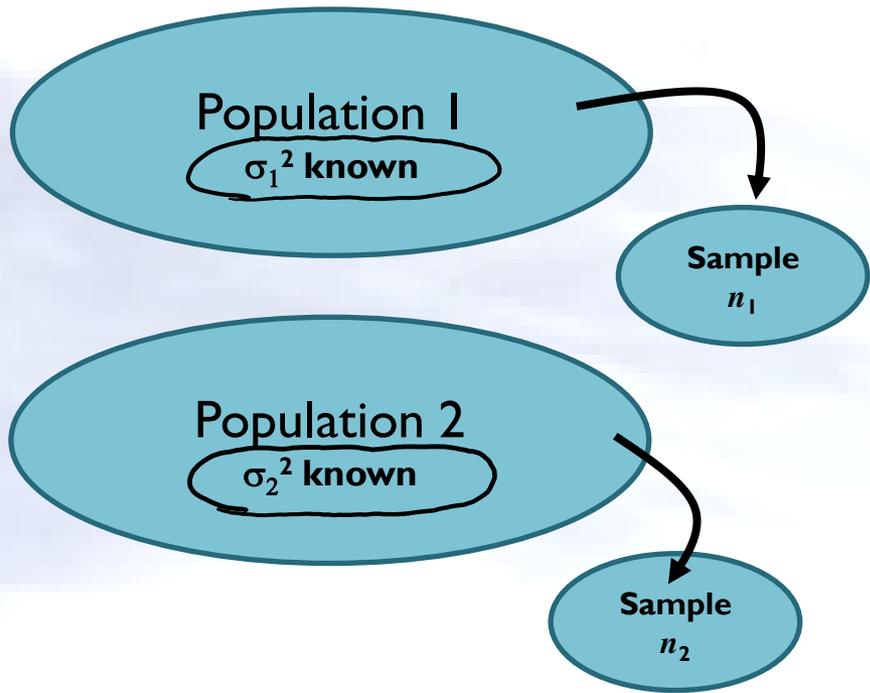
Sample n_2

$$Z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{(\sigma_1^2/n_1) + (\sigma_2^2/n_2)}}$$

$$\mathrm{E}\,(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$$

Stdev

$$\mathrm{Var}\,(\bar{x}_1 - \bar{x}_2) = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Difference in 2 sample means $(\bar{X}_1 - \bar{X}_2)$ must be normal to transform to Z

① $x_1 \sim \underline{N}$ and $x_2 \sim \underline{N}$ → $\bar{X}_1 \sim N$ $\bar{X}_2 \sim N$

Normal

$Z = \dfrac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$

OR

② n_1 & n_2 each ≥ 30 → $\bar{X}_1 \sim N$ $\bar{X}_2 \sim N$ by the CLT

$\bar{x}_1 - \bar{x}_2$ will be normally distributed because linear combination of normal variables is NORMALLY distributed

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Pooled Estimate of Variance

$T = \dfrac{\bar{X} - \mu}{S/\sqrt{n}}$

σ_1^2, σ_2^2 — Unknown

→ sample std dev.

❖ Use with T distribution

❖ When population variances are unknown believed equal $\sigma_1^2 = \sigma_2^2$, use **Pooled Estimate of Variance**

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

- Sample variances are pooled, or _Combined_
- S_p^2 is weighted average of two sample variances S_1^2 and S_2^2
  - weighted by degrees of freedom $n_1 - 1$ and $n_2 - 1$
- S_p^2 better estimator vs. using sample variances S_1^2 and S_2^2 individually

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