V14 - Sampling Distribution Theory - Part 9

Course: Statistical Testing & Regression
Dr. Renee Clark
Swanson School of Engineering
Industrial Engineering
University of Pittsburgh

Descriptive Statistics – Part 9

- T distribution (Student T)
- Compare T and Z distributions
- Properties of T distribution

t-Distribution

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- We're familiar with Z distribution
- Uses population standard deviation σ
- Knowing parameter value σ *might* be reasonable if highly familiar with population
- Generally, σ unknown
  - (If μ unknown, σ probably unknown too)
- However, as academic exercise/starting point, assumed σ known and used Z

t-Distribution

- Typically, must estimate σ using s
  - s is sample standard deviation
- Obtain random variable distributed according to T distribution:

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- T has $n-1$ degrees of freedom
- Parent population of X must be normally distributed to use T
  - $X \sim N$

t-distribution

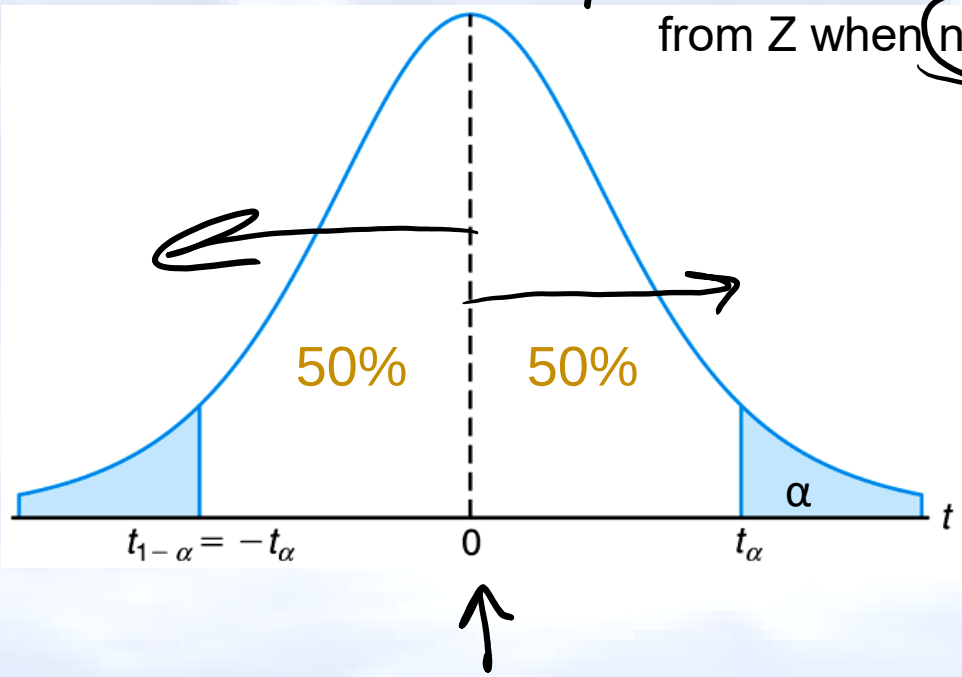
T looks like Z distribution!

→ As n → ∞,
Z and T become same distributions

T is symmetric about 0

→ T does not differ much from Z when n ≥ 30

Bell **Shaped**



t-Distribution

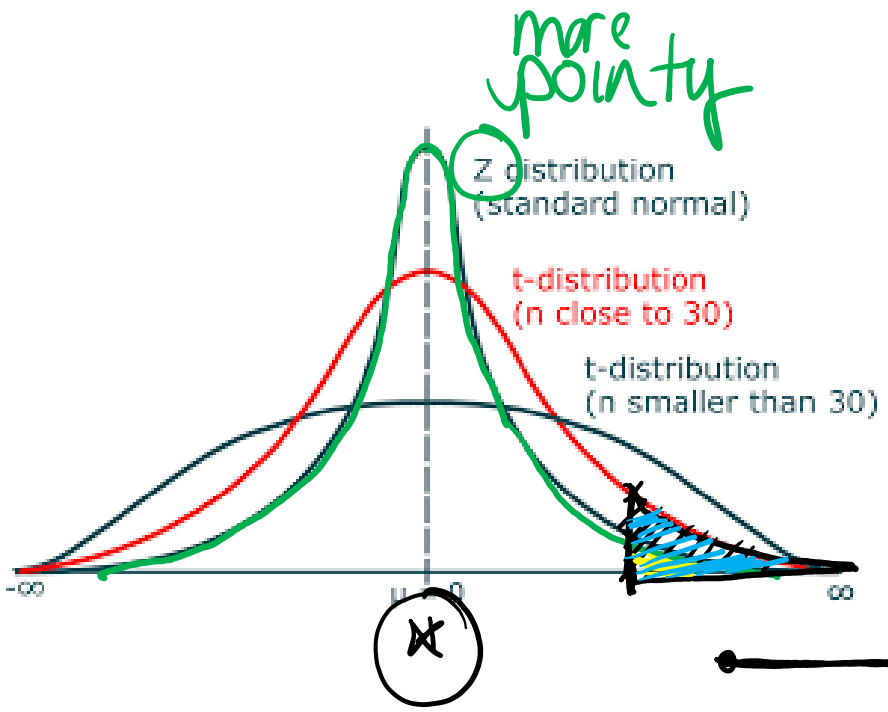
- More variable than Z distribution
- T depends on fluctuation of 2 statistics (random variables) from sample to sample:

$\bar{X}$ and S

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$$Z = \frac{\overline{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- CLT does not relate to T
- CLT relates to normal distribution (Z) and requires use of σ



T distribution less pointy than Z.

T has more area in tails.

T has heavier/thicker tails than Z.

Probability of getting value far from μ is greater with **T** (versus Z).

Acknowledgement

This material is based upon work partially supported by the National Science Foundation under Grant# 2335802. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

Under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon this work, you may distribute the resulting work only under the same or similar license to this one.

THE END