

VI4 - Sampling

Distribution Theory - Part 9

Course: Statistical Testing & Regression

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Descriptive Statistics – Part 9

- ❑ T distribution (Student T)
- ❑ Compare T and Z distributions
- ❑ Properties of T distribution



t-Distribution

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- We're familiar with Z distribution
- Uses population standard deviation
- Knowing parameter value _____ *might* be reasonable if highly familiar with population
- Generally, σ unknown
 - (If μ unknown, σ probably unknown too)
- However, as academic exercise/starting point, assumed σ known and used Z



t-Distribution

- Typically, must estimate σ using s
 - s is sample standard deviation
- Obtain random variable distributed according to T distribution:

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- T has _____ degrees of freedom
- ❖ Parent population of X must be _____ to use T
 - $X \sim$ _____



t-distribution

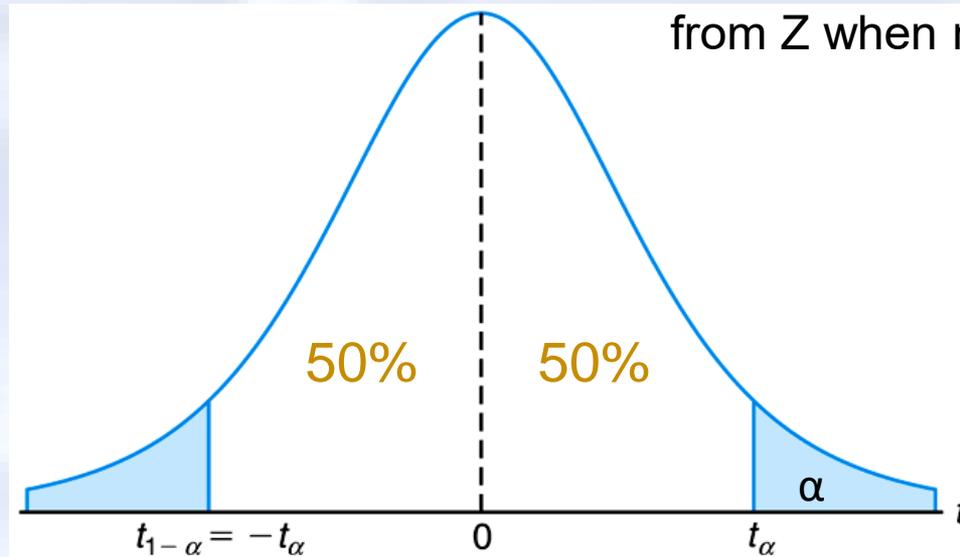
T looks like Z distribution!

As $n \rightarrow \infty$,
Z and T become _____

T is _____
about 0

T does not differ much
from Z when $n \geq 30$

_____ Shaped



t-Distribution

- More variable than Z distribution
- T depends on fluctuation of 2 statistics (random variables) from sample to sample:

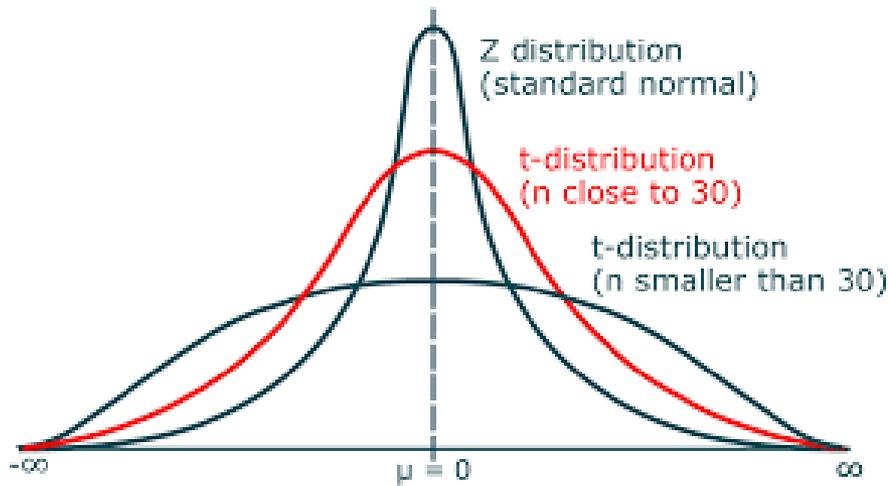
_____ and _____

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}}$$

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

- CLT does not relate to T
- CLT relates to normal distribution (Z) and requires use of σ





T distribution less pointy than Z.

T has more area in tails.

T has heavier/thicker tails than Z.

Probability of getting value far from μ is greater with **T** (versus Z).





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THE END

