

# Matrix Inverse

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## Reminder: Matrix form

- Matrix representation of linear system of equations

$$\begin{array}{l}
 a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = c_1 \\
 a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = c_2 \\
 \dots \\
 a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = c_m
 \end{array}
 \quad \longrightarrow \quad
 \begin{bmatrix}
 a_{11} & a_{12} & \cdot & \cdot & a_{1n} \\
 a_{21} & a_{22} & \cdot & \cdot & a_{2n} \\
 \vdots & & & & \vdots \\
 \vdots & & & & \vdots \\
 a_{m1} & a_{m2} & \cdot & \cdot & a_{mn}
 \end{bmatrix}
 \begin{bmatrix}
 x_1 \\
 x_2 \\
 \vdots \\
 \vdots \\
 x_n
 \end{bmatrix}
 =
 \begin{bmatrix}
 c_1 \\
 c_2 \\
 \vdots \\
 \vdots \\
 c_m
 \end{bmatrix}$$

- How to solve for X?

$$\begin{aligned}
 [A] \\
 [X] &= [C]
 \end{aligned}$$

## Can we divide two matrices?

- If  $[A][X] = [C]$  is defined, it might seem intuitive that  $[X] = \frac{[C]}{[A]}$ , but matrix division is not defined like that.
- The inverse of a square matrix  $[A]$ , if existing, is denoted by  $[A]^{-1}$  such that  $[A][A]^{-1} = [I] = [A]^{-1}[A]$ 
  - where  $[I]$  is the identity matrix.
- As a result, we can solve for  $[X]$  as follows

## Solution to a set of equations, $[A][X] = [C]$

- If the number of equations is the same as the number of unknowns, the coefficient matrix  $[A]$  is a square matrix.

Given

$$[A][X] = [C]$$

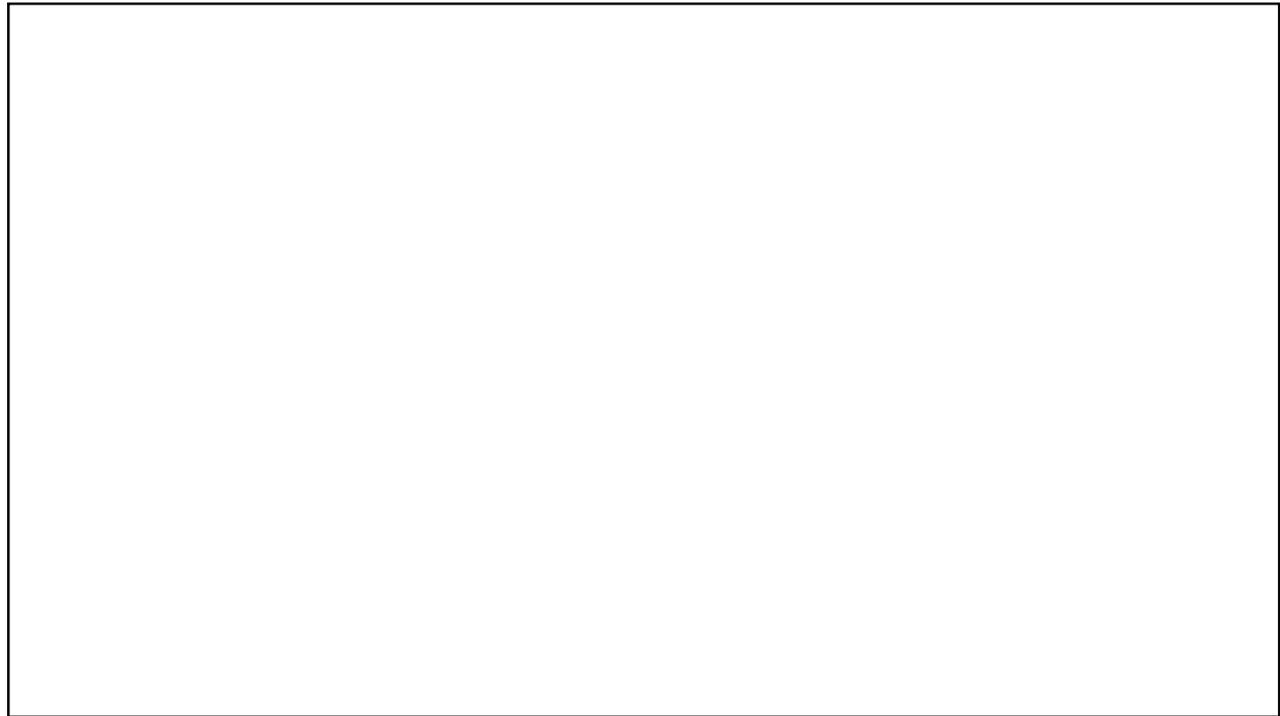
Then, if  $[A]^{-1}$  exists, multiplying both sides by  $[A]^{-1}$ .

$$[A]^{-1}[A][X] = [A]^{-1}[C]$$

$$[I][X] = [A]^{-1}[C]$$

$$[X] = [A]^{-1}[C]$$

This implies that if we are able to find  $[A]^{-1}$ , the solution vector of  $[A][X] = [C]$  is simply a multiplication of  $[A]^{-1}$  and the right-hand side vector,  $[C]$ .



## Matrix inverse

- Let  $A$  be a square matrix. If  $[B]$  is another square matrix of the same size such that  $[B] [A] = [I]$ , then  $[B]$  is the inverse of  $[A]$ .
  - $[A]$  is then called to be invertible or nonsingular.
- If  $[A]^{-1}$  does not exist,  $[A]$  is called noninvertible or singular.

## Matrix inverse

If  $[A]$  and  $[B]$  are two  $n \times n$  matrices such that  $[B][A] = [I]$ , then these statements are also true

- $[B]$  is the inverse of  $[A]$
- $[A]$  is the inverse of  $[B]$
- $[A]$  and  $[B]$  are both invertible
- $[A][B] = [I]$ .
- $[A]$  and  $[B]$  are both nonsingular
- all columns of  $[A]$  and  $[B]$  are linearly independent
- all rows of  $[A]$  and  $[B]$  are linearly independent.

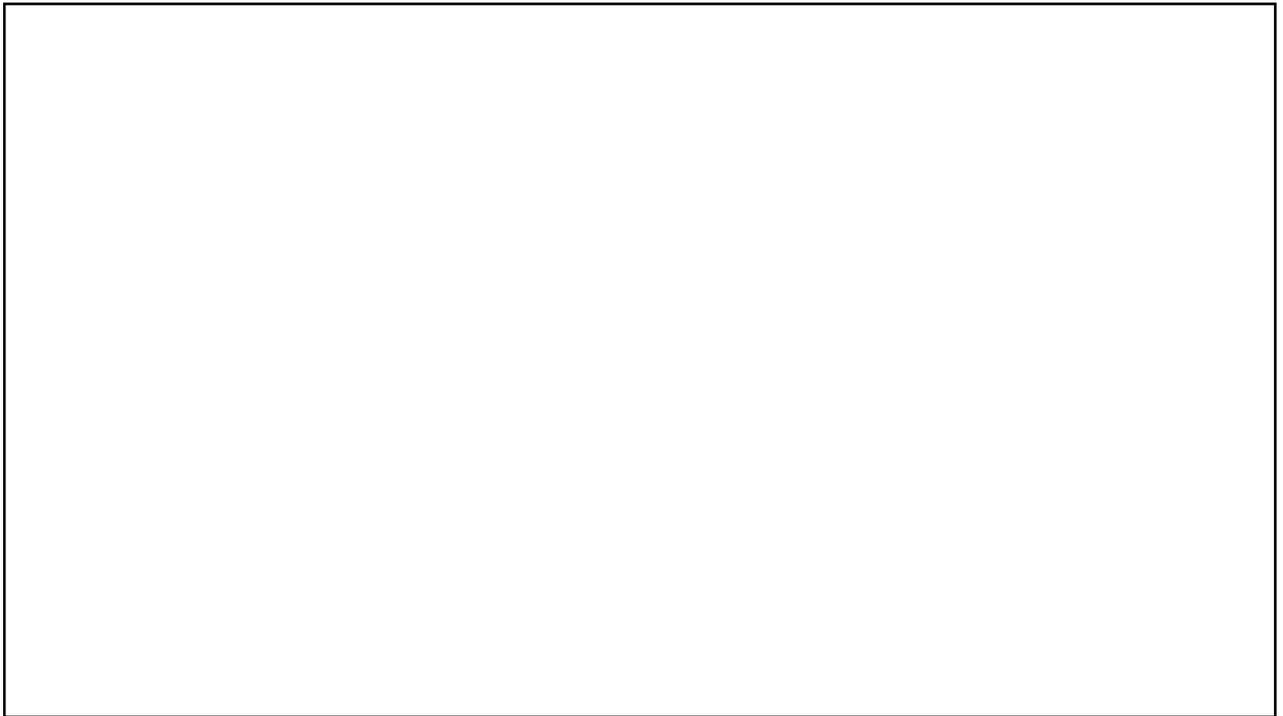
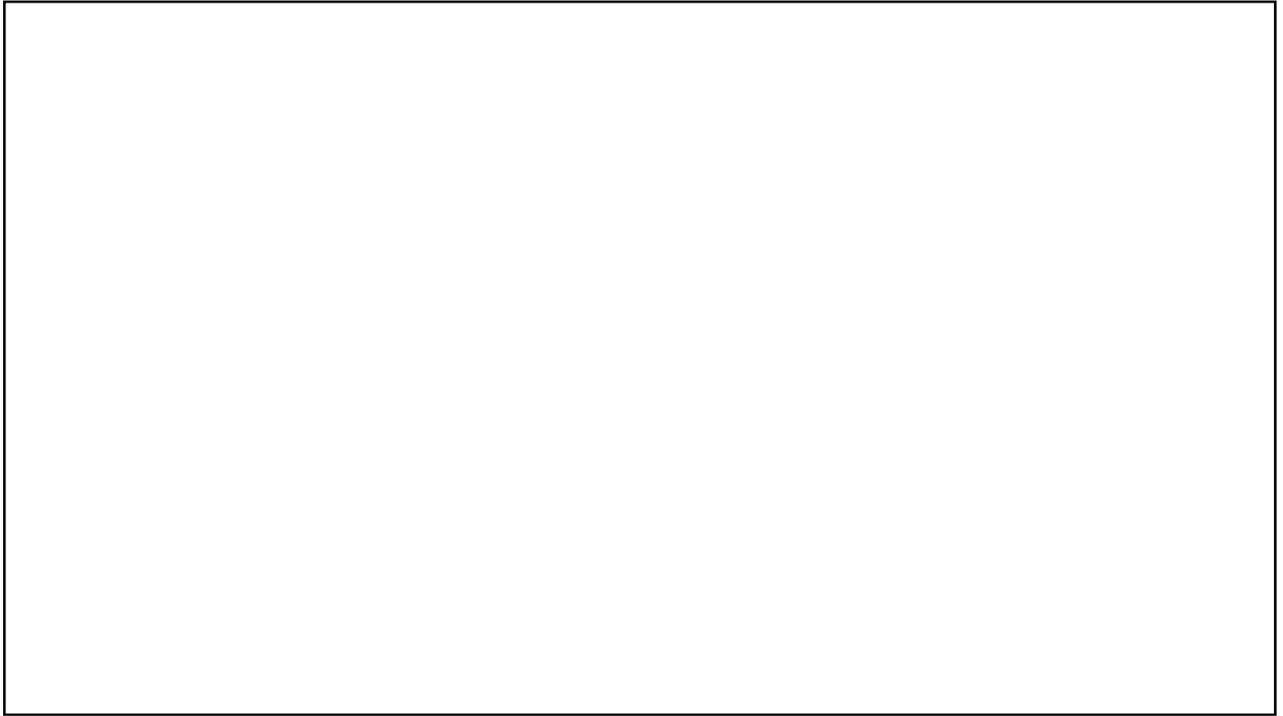
## Example

- Determine if

$$[B] = \begin{bmatrix} 3 & 2 \\ 5 & 3 \end{bmatrix}$$

is the inverse of

$$[A] = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$$



## Inverse of a 2x2 matrix

• Given

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

• Example

$$[A] = \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix}$$

Wrap it up: Solution  
of Simultaneous  
Equations using  
Matrix Method

## Example

- Solve the following system using the matrix method
  - $2x+y = 4$
  - $x-y = -1$

