Linear Circuits and Systems

Complex Numbers– Part 1 of 2

Course: Linear Circuits and Systems
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Objectives

This series will focus on:

- Review of Complex Numbers
  - Part 1:
    - The *Real* and *Imaginary* parts
    - Rectangular Form
    - Polar Form
    - Graphic Form
  - Part 2:
    - Arithmetic Operations
    - Examples

Complex Numbers

- In the set of real numbers, negative numbers do not have square roots.
- Imaginary numbers were invented so that negative numbers would have square roots and certain equations would have solutions.
- These numbers were devised using an **imaginary unit** named ***j***.
- Mathematicians use the symbol ***i*** for this number, but electrical engineers use ***j***:

$$i = \sqrt{-1} \quad \text{or} \quad j = \sqrt{-1}$$

A *complex number* is a number of the form $a + jb$, where a and b are real numbers and $j = \sqrt{-1}$.

The number a is the *real part* of $a + jb$, and b is its *imaginary part*.

REPRESENTATIONS OF COMPLEX NUMBERS

Rectangular Form
Polar Form
Exponential Form

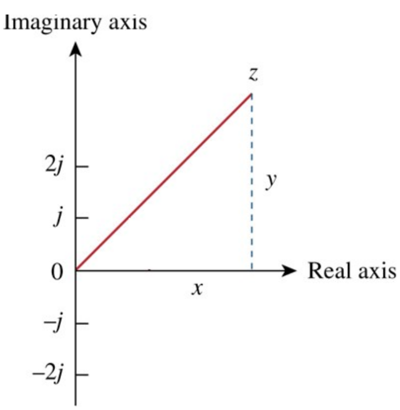
Rectangular Form

- In rectangular form, a complex number z is written as the sum of a **real part** x and an **imaginary part** y:

$$z = x + jy$$

The Complex Plane

- We often represent complex numbers as points in the complex plane, with the real part plotted along the horizontal axis (or "real axis") and the imaginary part plotted along the vertical axis (or "imaginary axis").

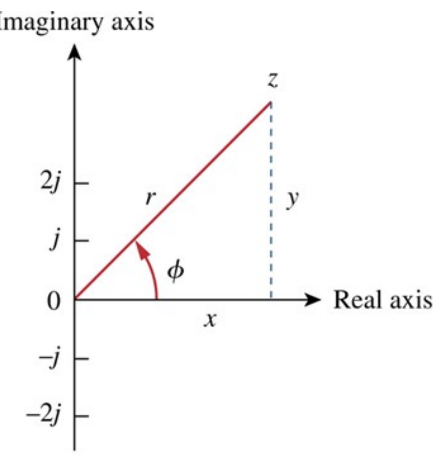


Polar Form

- In polar form, a complex number z is written as a **magnitude** *r* at an **angle** ϕ:

$$z = r\angle\phi$$

- The angle ϕ is measured from the positive real axis.



Converting from Rectangular Form to Polar Form

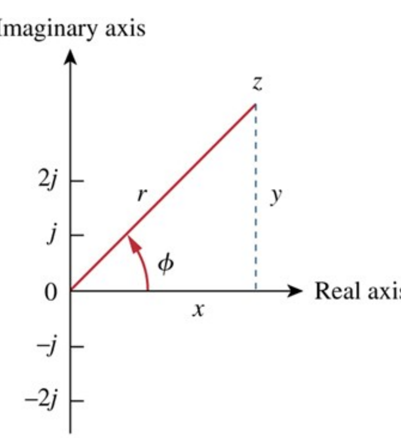
- Given a complex number *z* with real part *x* and imaginary part *y*, its magnitude is given by

$$r = \sqrt{x^2 + y^2}$$

and its angle is given by

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = r\angle\phi$$



Converting from Polar Form to Rectangular Form

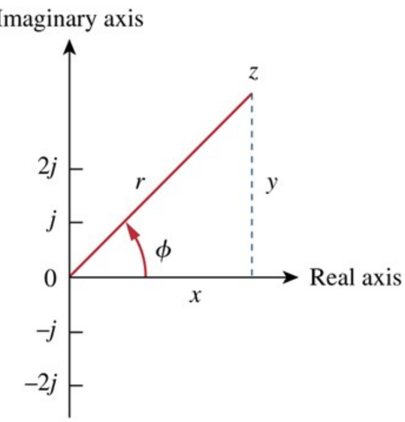
- Given a complex number z with magnitude ***r*** and angle ϕ, its real part is given by

$$x = r\cos\phi$$

and its imaginary part is given by

$$y = r\sin\phi$$

z = x + jy



Exponential Form

- Complex numbers may also be written in exponential form.

Polar form		Exponential Form
$r\angle\phi$	⇔	$r\,e^{j\phi}$

Example: $3\angle 30°$ ⇔ $3e^{\,j30}$

Euler’s Formula

- The exponential form is based on **Euler’s identity**, which says that, for any ϕ,

$$e^{j\phi} = \cos\phi + j\sin\phi$$

$$r\,e^{j\phi}$$

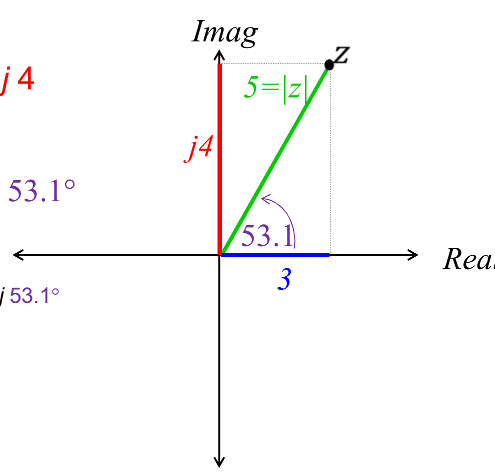
$$= r\,(\cos\phi + j\sin\phi)$$

$$= r\cos\phi + j\,r\sin\phi$$

$$= x + j\,y$$

Example

- Any complex number can be expressed in three forms:
  - Rectangular form
    - Example: z = 3 + *j* 4
  - Polar form
    - Example: z = 5 $\angle$ 53.1°
  - Exponential form
    - Example: z = 5 $e^{j\,53.1°}$



- Conclusion

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