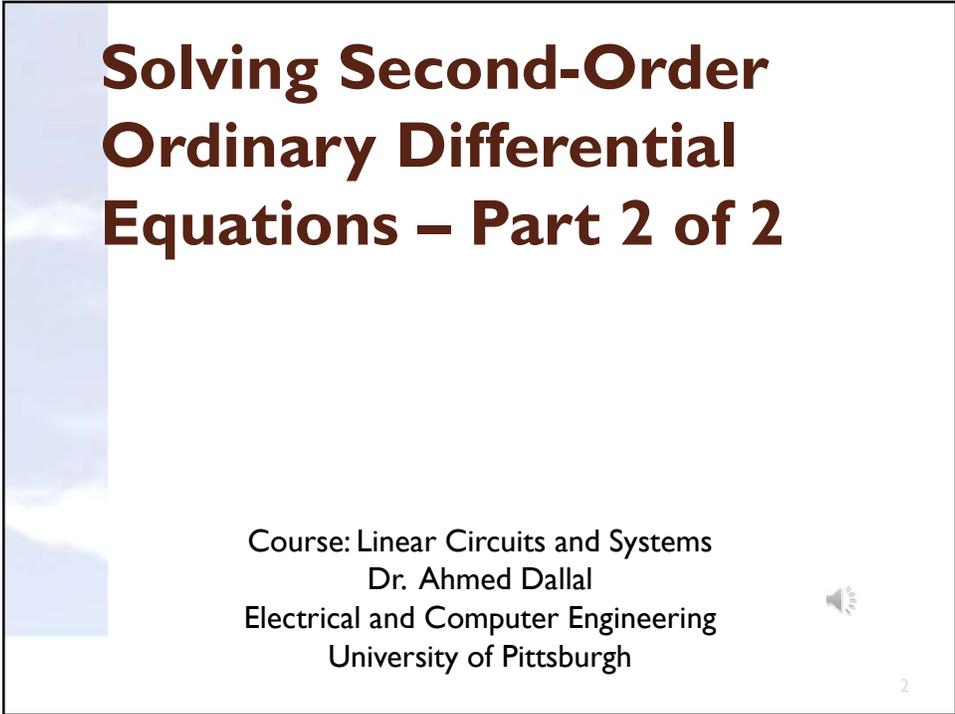




Linear Circuits and Systems



**Solving Second-Order
Ordinary Differential
Equations – Part 2 of 2**

Course: Linear Circuits and Systems
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Example

 $y(t)$

- Solve

$$2 \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 3.125y = \sin t,$$

$$y(0) = 5, \quad \frac{dy}{dt}(t=0) = 3$$

$$y(t) = y_H(t) + y_P(t)$$

Homogenous Solution

$$2s^2 + 3s + 3.125 = 0$$

$$\alpha_1, \alpha_2 = \frac{-3 \pm \sqrt{9 - 4 \times 2 \times 3.125}}{2 \times 2}$$

$$= -0.75 \pm j1$$

$$\alpha \quad \beta$$

$$y_H(t) = e^{\alpha t} (k_1 \cos \beta t + k_2 \sin \beta t)$$

$$y_H(t) = e^{-0.75t} (k_1 \cos t + k_2 \sin t)$$

Particular Solution

$$2 \frac{d^2 y}{dt^2} + 3 \frac{dy}{dt} + 3.125y = \sin t$$

$$y_p(t) = A \sin t + B \cos t$$

$$\frac{dy_p(t)}{dt} = A \cos t - B \sin t$$

$$\frac{d^2 y_p(t)}{dt^2} = -A \sin t - B \cos t$$

Particular Solution

$$2(-A \sin t - B \cos t) + 3(A \cos t - B \sin t)$$

$$+ 3.125(A \sin t + B \cos t) = \sin t$$

$$(1.125A - 3B) \sin t +$$

$$(1.125B + 3A) \cos t = 1 \sin t$$

$$1.125B + 3A = 0 \rightarrow (1)$$

$$1.125A - 3B = 1 \rightarrow (2)$$

$$A = 0.109, \quad B = -0.292$$

$$y_p = 0.109 \sin t - 0.292 \cos t$$

$$y(t) = y_H + y_p$$

$$= e^{-0.75t} (k_1 \cos t + k_2 \sin t)$$

$$+ 0.109 \sin t - 0.292 \cos t$$

$$y(0) = 5 ; \left[\frac{dy}{dt} (t=0) = 3 \right]$$

$$5 = e^{-0.75(0)} (k_1 \cos 0 + k_2 \sin 0)$$

$$+ 0.109 \sin 0 - 0.292 \cos 0$$

$$5 = k_1 - 0.292 \rightarrow k_1 = 5.292$$

• Complete exact solution

$$\frac{dy}{dt} = -0.75e^{-0.75t} (k_1 \cos t + k_2 \sin t)$$

$$+ e^{-0.75t} (-k_1 \sin t + k_2 \cos t)$$

$$+ 0.109 \cos t + 0.292 \sin t$$

$$\frac{dy}{dt} \Big|_{t=0} = 3 = -0.75e^{-0} (k_1 \cos 0 + k_2 \sin 0)$$

$$+ e^0 (-k_1 \sin 0 + k_2 \cos 0) + 0.109 \cos 0 + 0.292 \sin 0$$

$$3 = -0.75 k_1 + k_2 + 0.109$$

$$3 = -0.75(5.529) + k_2 + 0.109$$

$$k_2 = 6.859$$

the Complete Soluⁿ

$$y(t) = e^{-0.75t} (5.292 \cos t + 6.859 \sin t) + 0.109 \sin t - 0.292 \cos t$$

#



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