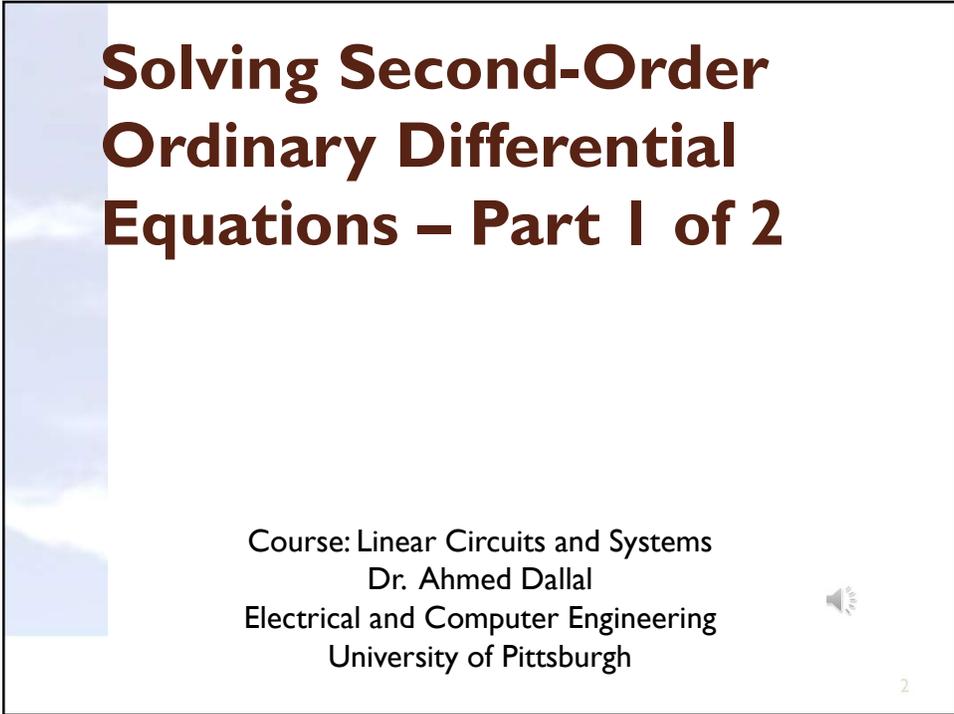




Linear Circuits and Systems



**Solving Second-Order  
Ordinary Differential  
Equations – Part I of 2**

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## 2<sup>nd</sup> Order Linear ODE

- The general form of a second-order linear ordinary differential equation with constant coefficients is given by

$$\rightarrow k_2 \frac{d^2 y}{dt^2} + k_1 \frac{dy}{dt} + k_0 y = F(t) \leftarrow$$

$y(t)$

- The general solution contains two parts

$$y = y_H + y_P$$

## Homogenous Solution

- The homogeneous part of the solution  $y_H$  is that part of the solution that gives **zero** when substituted in the left-hand side of the ODE.

$$\rightarrow k_2 \frac{d^2 y_H}{dt^2} + k_1 \frac{dy_H}{dt} + k_0 y_H = 0 \leftarrow$$

- Derivatives of exponentials are Exponentials too!

## Homogenous Solution – Characteristic Polynomial

$$k_2 \frac{d^2 y_H}{dt^2} + k_1 \frac{dy_H}{dt} + k_0 y_H = 0$$

- reduced the differential equation to an ordinary quadratic equation

$$\rightarrow k_2 s^2 + k_1 s + k_0 = 0 \leftarrow$$

- The roots of this equation determines the homogenous solution.

## Homogenous Solution – Characteristic Polynomial

$$\rightarrow k_2 s^2 + k_1 s + k_0 = 0 \leftarrow$$

- The roots of the characteristic equation are

$$\alpha_1, \alpha_2 = \frac{-k_1 \pm \sqrt{k_1^2 - 4k_2 k_0}}{2k_2}$$

## Homogenous Solution – Characteristic Polynomial

- Based on the nature of the roots, we have three possibilities of the homogenous solution:

1. 2 unique real roots

$$y_H = \underline{K_1} e^{\alpha_1 t} + \underline{K_2} e^{\alpha_2 t}$$

2. 2 identical roots ( $\alpha_1 = \alpha_2 = \alpha$ )

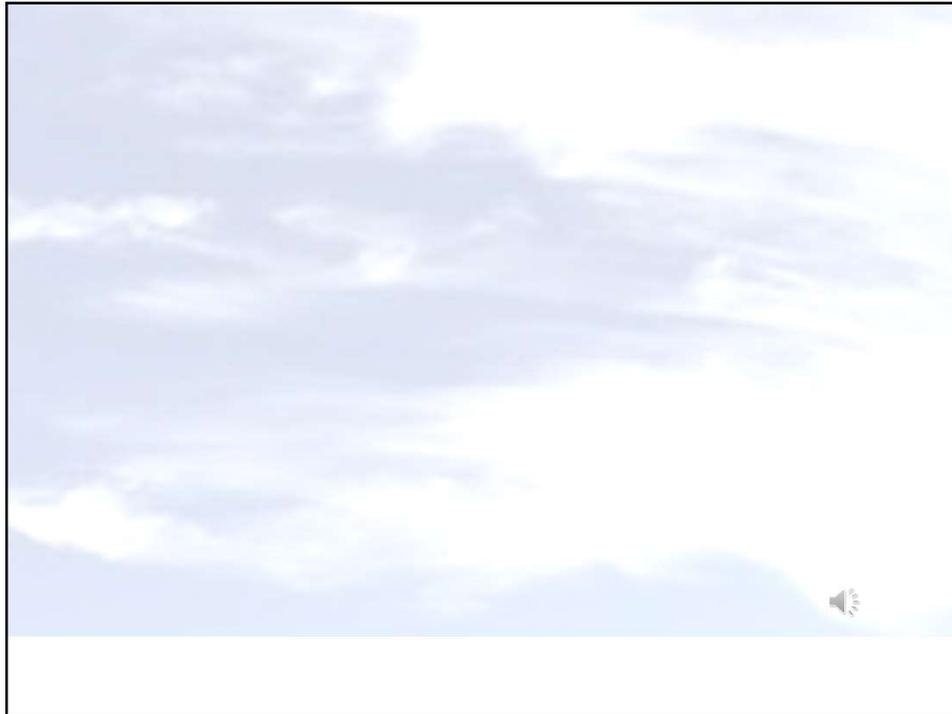
$$y_H = \underline{K_1} e^{\alpha t} + \underline{K_2} t e^{\alpha t}$$

## Homogenous Solution – Characteristic Polynomial

- Based on the nature of the roots, we have three possibilities of the homogenous solution:

3. 2 unique complex roots ( $\alpha_1, \alpha_2 = \alpha \pm j\beta$ )

$$y_H = \underline{e^{\alpha t}} (\underline{K_1} \cos(\beta t) + \underline{K_2} \sin(\beta t))$$



## Particular Solution

- The particular part of the solution  $y_p$  is that part of the solution that gives  $F(t)$  when substituted for  $y$  in the ODE.
- Usually the particular solution is similar in form to  $F(t)$ .

## Particular Solution

$$k_2 y'' + k_1 y' + k_0 y = F(t)$$

$F(t)$	$y_p(t)$
$a_0$	$b_0$
$a_0 + a_1 t + a_2 t^2$	$b_0 + b_1 t + b_2 t^2$
$e^{at}$	$Ae^{ax}$
$\sin(bt)$	$A\sin(bt) + B\cos(bt)$
$e^{at}\sin(bt)$	$e^{at}(A\sin(bt) + B\cos(bt))$
$\cos(bt)$	$A\sin(bt) + B\cos(bt)$
$e^{at}\cos(bt)$	$e^{at}(A\sin(bt) + B\cos(bt))$

## Conclusion



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