Linear Circuits and Systems

Solving Second-Order Ordinary Differential Equations – Part 1 of 2

Course: Linear Circuits and Systems
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2^{nd} Order Linear ODE

- The general form of a second-order linear ordinary differential equation with constant coefficients is given by

$$k_2 \frac{d^2 y}{dt^2} + k_1 \frac{dy}{dt} + k_0 y = F(t)$$

- The general solution contains two parts

$$y = y_H + y_P$$

Homogenous Solution

- The homogeneous part of the solution y_H is that part of the solution that gives **zero** when substituted in the left-hand side of the ODE.

$$k_2 \frac{d^2 y_H}{dt^2} + k_1 \frac{dy_H}{dt} + k_0 y_H = 0$$

- Derivatives of exponentials are Exponentials too!

Homogenous Solution – Characteristic Polynomial

$$k_2 \frac{d^2 y_H}{dt^2} + k_1 \frac{dy_H}{dt} + k_0 y_H = 0$$

- reduced the differential equation to an ordinary quadratic equation

$$k_2 s^2 + k_1 s + k_0 = 0$$

  - The roots of this equation determines the homogenous solution.

Homogenous Solution – Characteristic Polynomial

$$k_2 s^2 + k_1 s + k_0 = 0$$

- The roots of the characteristic equation are

$$\alpha_1, \alpha_2 = \frac{-k_1 \pm \sqrt{k_1^2 - 4k_2 k_0}}{2k_2}$$

Homogenous Solution – Characteristic Polynomial

- Based on the nature of the roots, we have three possibilities of the homogenous solution:

1. 2 unique real roots
$$y_{\mathrm{H}} = K_1 e^{\alpha_1 t} + K_2 e^{\alpha_2 t}$$

2. 2 identical roots ($\alpha_1 = \alpha_2 = \alpha$)
$$y_{\mathrm{H}} = K_1 e^{\alpha t} + K_2 t e^{\alpha t}$$

Homogenous Solution – Characteristic Polynomial

- Based on the nature of the roots, we have three possibilities of the homogenous solution:

3. 2 unique complex roots ($\alpha_1, \alpha_2 = \alpha \pm j\beta$)
$$y_{\mathrm{H}} = e^{\alpha t}(K_1 \cos(\beta t) + K_2 \sin(\beta t))$$

Particular Solution

- The particular part of the solution y_P is that part of the solution that gives $F(t)$ when substituted for y in the ODE.

- Usually the particular solution is similar in form to $F(t)$.

Particular Solution

$F(t)$	$y_P(t)$
a_0	b_0
$a_0 + a_1 t + a_2 t^2$	$b_0 + b_1 t + b_2 t^2$
e^{at}	Ae^{ax}
$\sin(bt)$	$A\sin(bt) + B\cos(bt)$
$e^{at}\sin(bt)$	$e^{at}\left(A\sin(bt) + B\cos(bt)\right)$
$\cos(bt)$	$A\sin(bt) + B\cos(bt)$
$e^{at}\cos(bt)$	$e^{at}\left(A\sin(bt) + B\cos(bt)\right)$

- Conclusion

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