Linear Circuits and Systems

Solving First-Order Ordinary Differential Equations – Part 2 of 2

Course: Linear Circuits and Systems
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Example 1

- Solve the ordinary differential equation exactly

$$3\frac{dy}{dt} + 2y = e^{-t},\ y(0) = 5$$

$k_1\, dy/dt + k_0 y = F(t)$

$y(t) = y_H(t) + y_p(t)$

$$y_H(t) = K e^{-k_0/k_1 t} = K e^{-2/3 t}$$

$$y_p(t) = A e^{-t} \qquad F(t) = e^{-t}$$

$$3\frac{dy_p}{dt} + 2y_p = e^{-t}$$

$$3(-Ae^{-t}) + 2Ae^{-t} = e^{-t}$$

$$(-3A + 2A)e^{-t} = e^{-t}$$

$$-3A + 2A = 1 \Rightarrow \boxed{A = -1}$$

$$y_p = -e^{-t}$$

$$y(t) = ke^{-2/3\,t} - e^{-t}$$

$$y(0) = 5$$

$$y(0) = ke^{-0} - e^{-0} = 5$$

$$k - 1 = 5 \Rightarrow \boxed{k = 6}$$

$$\boxed{y(t) = 6e^{-2/3\,t} - e^{-t}}$$

Example 2

$y(x)$

- Solve the ordinary differential equation exactly

$$\frac{dy}{dx} + 2y = 50,\ y(0) = 40$$

$k_1 \frac{dy}{dx} + k_0 y = F(x)$

$y(x) = y_H + y_P$

$y_H = K e^{-k_0/k_1 x} = K e^{-2x}$

$y_p(x) = b_0$ $\qquad f(x) = 50$

$$\frac{dy_p}{dx} + 2y_p = 50$$

$$0 + 2b_0 = 50$$

$$\Rightarrow b_0 = 25$$

$$\boxed{y_p(x) = 25}$$

$\rightarrow y(x) = k e^{-2x} + 25 \leftarrow$

$y(0) = 40$

$y(0) = k e^{-2(0)} + 25 = 40$

$k e^{0} = 1$

$\boxed{k = 15}$

$\boxed{y(x) = 15 e^{-2x} + 25}$ #

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