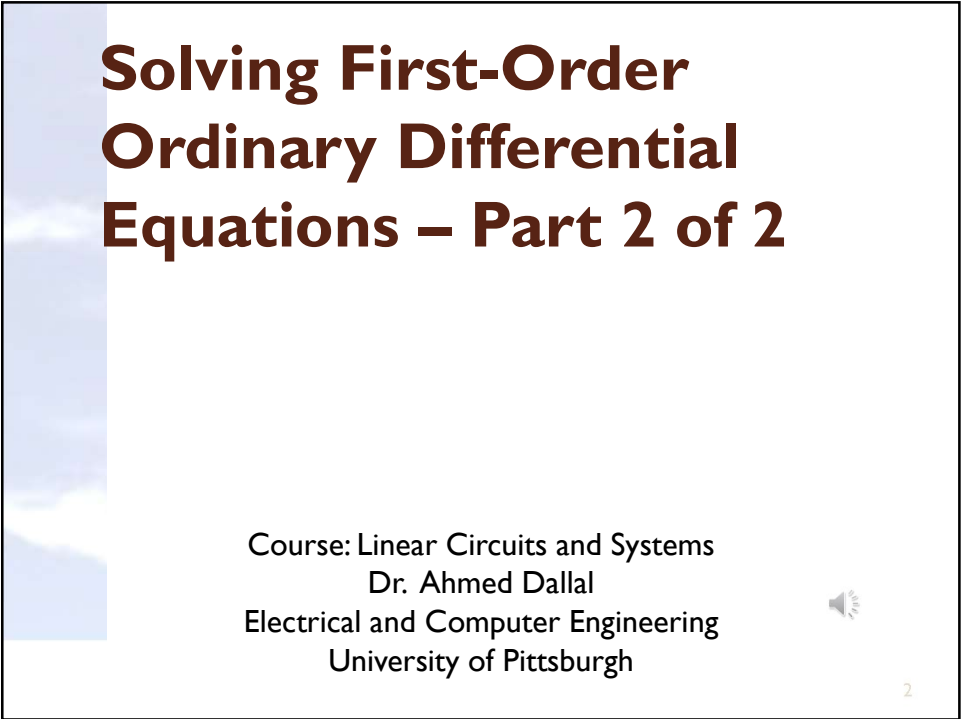




Linear Circuits and Systems



Solving First-Order Ordinary Differential Equations – Part 2 of 2

Course: Linear Circuits and Systems
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Example I

- Solve the ordinary differential equation exactly

$$3 \frac{dy}{dt} + 2y = e^{-t}, y(0) = 5$$

$$(k_1) \frac{dy}{dt} + (k_0)y = F(t)$$

$$y(t) = y_H(t) + y_P(t)$$

$$y_H(t) = K e^{-k_0/k_1 t} = K e^{-2/3 t}$$

$$y_P(t) = A e^{-t}$$

$$F(t) = e^{-t}$$

$$3 \frac{dy_P}{dt} + 2y_P = e^{-t}$$

$$3(-A e^{-t}) + 2A e^{-t} = e^{-t}$$

$$\rightarrow (-3A + 2A) e^{-t} = e^{-t} \leftarrow$$

$$-3A + 2A = 1 \Rightarrow \boxed{A = -1}$$

$$y_p = -e^{-t}$$

$$y(t) = k e^{-2/3 t} - e^{-t}$$

$$y(0) = 5$$

$$y(0) = k e^{-0} - e^{-0} = 5$$

$$k - 1 = 5 \Rightarrow \boxed{k = 6}$$

$$y(t) = 6 e^{-2/3 t} - e^{-t}$$



Example 2 $y(x)$

- Solve the ordinary differential equation exactly

$$\frac{dy}{dx} + 2y = 50, y(0) = 40$$

$$(k_1) \frac{dy}{dx} + (k_0)y = F(x)$$

$$y(x) = y_H + y_P$$

$$y_H = k e^{-k_0/k_1 x} = k e^{-2x}$$

$$y_p(x) = b_0$$

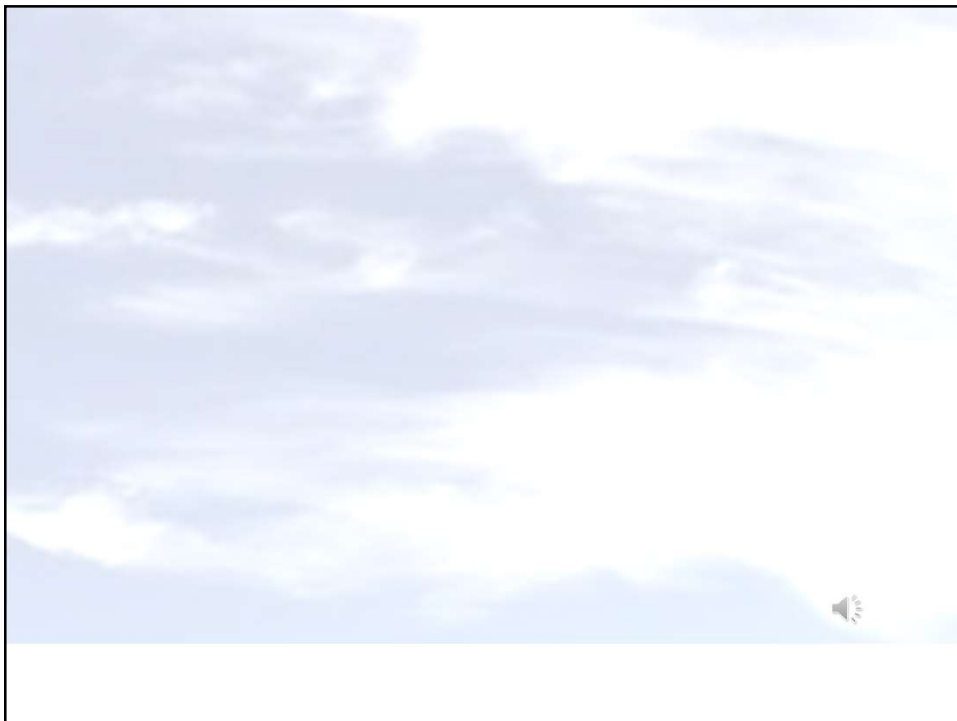
$$f(x) = 50$$

$$\frac{dy_p}{dx} + 2y_p = 50$$

$$0 + 2b_0 = 50$$

$$\Rightarrow b_0 = 25$$

$$\boxed{y_p(x) = 25}$$



$$\rightarrow y(x) = K e^{-2x} + 25 \leftarrow$$

$$y(0) = 40$$

$$y(0) = K e^{\cancel{0}} + 25 = 40$$

$\underbrace{\quad\quad\quad}_{[K=15]}$

$$\boxed{y(x) = 15 e^{-2x} + 25}$$



#





Acknowledgement

This material is based upon work partially supported by the National Science Foundation under Grant# 2335802. Any opinions, findings, and conclusions, or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.



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