Linear Circuits and Systems

Solving First-Order Ordinary Differential Equations – Part 1 of 2

Course: Linear Circuits and Systems
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1st Order Linear ODE

- The general form of a first-order linear ordinary differential equation with constant coefficients is given by

y(t)

$$k_1 \frac{dy}{dt} + k_0 y = F(t)$$

$$k_1 y' + k_0 y = F(t)$$

- The general solution contains two parts

$$y = y_H + y_P$$

Homogenous Solution

- The homogeneous part of the solution y_H is that part of the solution that gives **zero** when substituted in the left-hand side of the ODE.

$$\boxed{k_1 \frac{dy_H}{dt} + k_0 y_H = 0}$$

Homogenous Solution

$$k_1 \frac{dy_H}{dt} + k_0 y_H = 0$$

$$k_1 \frac{dy_H}{dt} = -k_0 y_H$$

$$\int \frac{dy_H}{y_H} = \int \frac{-k_0}{k_1} dt$$

$$\ln y_H = \frac{-k_0}{k_1} t + C$$

$$\ln y_H = -\frac{k_0}{k_1} t + C$$

$$y_H = e^{-\frac{k_0}{k_1} t + C}$$

$$= e^{-k_0/k_1 t} \cdot e^C$$

$$\boxed{y_H(t) = K e^{-k_0/k_1 t}}$$

Particular Solution

- The particular part of the solution y_P is that part of the solution that gives $F(t)$ when substituted for y in the ODE.
- Usually the particular solution is similar in form to $F(t)$.

Particular Solution

$$K_g \frac{dy_P}{dt} + y_P = F(t)$$

g

$F(t)$	$y_P(t)$
a_0	b_0
$a_0 + a_1 t + a_2 t^2$	$b_0 + b_1 t + b_2 t^2$
e^{at}	Ae^{ax}
$\sin(bt)$	$A\sin(bt) + B\cos(bt)$
$e^{at}\sin(bt)$	$e^{at}(A\sin(bt) + B\cos(bt))$
$\cos(bt)$	$A\sin(bt) + B\cos(bt)$
$e^{at}\cos(bt)$	$e^{at}(A\sin(bt) + B\cos(bt))$

Conclusion

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