Linear Circuits and Systems

Absolute Maximum of a Function

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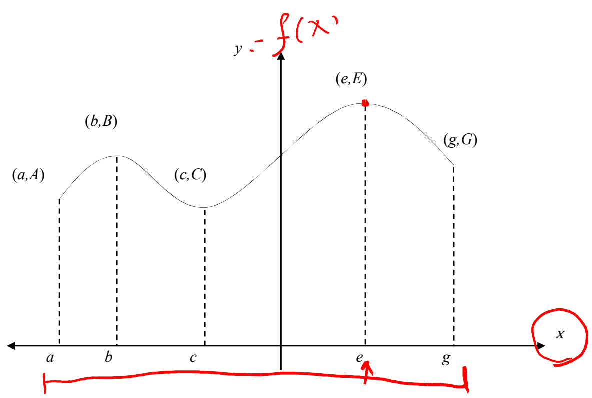
Introduction

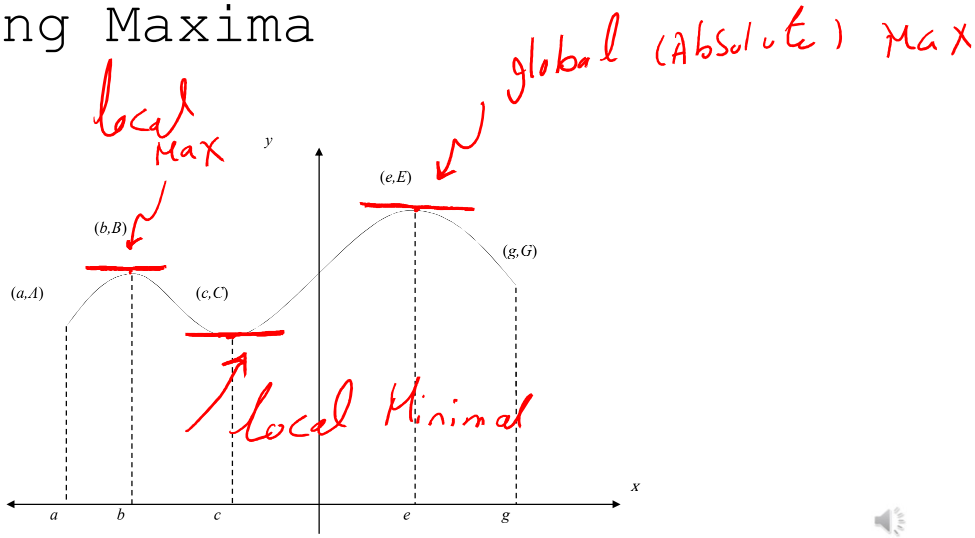
- In circuit analysis, we often want to find the value of a circuit parameter that maximize power transfer.
  - We can write the power as a function of this circuit parameter.
- In this primer, we will cover the basics of finding the maximum of a continuous function that is twice differentiable with domain D.

Absolute Minimum Value

- Given a function $f(x)$ with domain D, then $f(e)$ is the absolute maximum on D if and only if $f(x) \leq f(e)$ for all x in D



Finding Maxima



Finding Maxima

- To find the absolute maximum of a continuous function with domain D, we
  - look at the value of the function at the endpoints of D and also
  - check where $\boxed{f'(x) = 0.}$
- These points where $f'(x) = 0$ are the points of local extreme (local minimum or maximum) values.
  - If $f''(x) < 0$ at any of these points where local extremes occur, then it corresponds to a local maximum.

Finding the absolute maximum

- Out of all the local Maximas and the values at the domain ends, one can find the maximum of such values.
- The point where this maximum exists is then the location of the absolute maximum value, and the value of the function at that point is the absolute maximum.

Summary

Step 1: find all candidates for local extrima, x_i such that $f'(x_i) = 0$.

Step 2: identify which candidates corresponds to a maximum, $f''(x_i) < 0$.

Step 3: Compute $f(x)$ at the locations corresponding to maximum

Step 4: Compute $f(x)$ at the end of its domain

Step 5: Identify the location of the absolute maximum and its value

Example

- Find the location of the absolute maximum of a polynomial $(-\infty, \infty)$
$$f(x) = 25 + 20x - 4x^2.$$

$f'(x) = 20 - 8x = 0 \rightarrow x = \frac{20}{8} = \frac{5}{2}$

Maxima

$f''(x) = -8$

$f(x = 5/2) = 50$

$f(-\infty) = -\infty$, $f(\infty) = -\infty$

is @ max value of 50 when $x = 5/2$

Example

- For the function $f(x) = 3x - x^3$, evaluate the absolute max and its location on the interval $[-2, 1.5]$

$\circledast$ $f'(x) = 3 - 3x^2 = 0$ $\quad x = -1$, $x = +1$

$\circledast$ $f''(x) = -6x$ $\quad f''(-1) = 6$, $f''(+1) = -6 < 0$

NOT Maxima $\quad$ Maxima

$\circledast$ $f(x = +1) = 3 - (1)^3 = 2$

$\circledast$ $f(-2) = -2 * 3 - (-2)^3 = -14$ $\quad * f(1.5) = 3 * 1.5 - (1.5)^3 = 1.125$

$\boxed{\text{Max } f(x) \text{ is } 2 \text{ @ } x = +1}$

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