

Hey everyone, welcome to Engineering Earth. In this video we're going to review the calculation of Reynolds number and discuss the difference in Reynolds number calculations for internal and free surface flows.

We've calculated many a Reynolds number by this time in the course, so of course we understand now that Reynold's number is a dimensionless parameter that expresses the ratio of inertial and viscous forces within fluid flows. Uh, and is governed by these equations where ρ is the fluid's density, U is the velocity of flow, L is a length scale, that's characteristic of our problem's geometry and μ and ν respectively are uh the dynamic and kinematic viscosities of the fluid. So when the numerator is large relative to the denominator this tells us that inertial forces are large enough to break out of their viscous restraints, and turbulent fluctuations can occur within the flow. On the other hand, um when the viscous forces on the denominator are relatively large they're then able to constrain those relatively smaller inertial forces that create the turbulent fluctuations, and then the flow quality is laminar. Everything stays nice and streamlined um and parallel, ordered flow. We're going to focus now on um this parameter L up here in our Reynolds number calculation, the characteristic length scale. So for many weeks now we've been calculating Reynolds numbers of internal pipe flows, and everyone is really accustomed now to thinking about this characteristic length scale, L , as D , the diameter of a pipe. But the length scale that we use in calculating the Reynolds number actually varies depending on the geometry of the problem. So the question we should ask ourselves when we're choosing the right length scale of a problem is: what size is the very largest turbulent eddy that is physically possible to form in my problem? So for internal pipe flow any turbulent eddy that's larger than the pipe diameter would be constrained by the pipe walls So a pipe diameter is a good length scale to choose What do I mean when I say this? Well let's say that I have a circular pipe cross-section like this. And then my flow area is the internal pipe cross-section like this. And these black lines are my physical boundaries, the pipe wall. So my characteristic length scale is going to be the size of fluid flow within these physical boundaries that that is going to constrain the size of turbulent eddies that can develop. So actually the diameter of the pipe um is is another parameter called the hydraulic diameter of the flow. And this hydraulic diameter is defined as four times the cross-sectional area of flow divided by the wetted perimeter of the flow, which is given by this variable P . And so the wetted perimeter of flow is the length of the flow that touches the walls of the pipe, like this. So for a circular pipe section my hydraulic radius is going to be four times the cross-sectional flow area, which is going to be $\pi D^2/4$ divided by my wetted perimeter which is just going to be πD . So everything here is going to cancel, leaving me just with the pipeline diameter. So for a circular cross-section the

hydraulic diameter is the same length as the diameter of the pipe. And after we calculate the Reynolds number of a pipe flow, then we have to interpret any number. Um so we know that any number that is about 2,000, 2300 or less we call this a laminar flow. And anything that's greater than 2300 we consider transitional and eventually fully turbulent.

So now let's think about a different problem geometry for calculating a Reynolds number. Imagine now we have a liquid that's flowing in a channel with an air liquid interface, otherwise known as a free surface flow. So just like water flowing in a river right, something like this. Our channel width is this dimension B and the depth of the liquid in our channel is Y . So again if I think of the largest turbulent eddy that could form in this situation, it's going to be constrained by the depth of the liquid in the channel. So rivers are usually a lot wider than they are deep, so this y dimension is usually going to be the one that's limiting the scale of turbulence that could develop. So my length scale to use in calculating a Reynolds number um might be kind of similar to the depth of the flow. So let's do the same thing that we did for an internal flow, let's calculate the hydraulic diameter of um of a free surface flow like this one. So again it's four times my cross-sectional flow area divided by my wetted perimeter. And so for a rectangular flow area, like the one that we have here, my cross-sectional flow area is just going to be B multiplied by Y . And my wetted perimeter, again this is going to be all the length over which a uh the water is touching a a solid. So kind of right along this this red dashed line that we can see here, that will be our wetted perimeter. And so if I want to add up that length my wetted perimeter of my open channel this is just going to be $2y$ plus b . So then putting these into my hydraulic diameter I get this. But I'm not going to use the hydraulic diameter to calculate the Reynolds number in my free surface flow. I'm just going to be a little bit more difficult I'm going to use something different I'm going to use the hydraulic radius. And my hydraulic radius is just cross-sectional flow area divided by my wetted perimeter. So for my rectangular flow this is just going to be b divided by $2y$ plus b . And if I make a comparison of my hydraulic radius and my hydraulic diameter I can see that they are similar to one another but that they vary by a factor of four. Right, my hydraulic diameter is four times the cross-sectional area divided by the wetted perimeter, whereas my hydraulic radius is just cross-sectional area divided by wetted perimeter. So I can say that my hydraulic diameter is equal to four times my hydraulic radius. And you might be thinking well that's not fair we're always going to calculate a bigger Reynolds number in an internal flow because we're using the hydraulic diameter as the length scale. And in a free surface flow we're using one quarter of the hydraulic diameter as the length scale. And you know what you would be absolutely spot on about that. So what are we going to do we have to level out the playing field. And we're going to do it in the way that we interpret the Reynolds number of a free surface flow. So

after I calculate a Reynolds number for a free surface flow, if that Reynolds number is less than 500 I'm calling that laminar. If it goes above 500 it becomes transitional and turbulent. And you remember in an internal flow we had a different standard of what it was to be turbulent versus laminar here. Anything less than 2300 was laminar flow. Why do we interpret them different? Why does it take four times as large of a Reynolds number to get to a transitional or turbulent flow as compared to a laminar flow? Um well it's because in one we're using the hydraulic radius and in the other we're using the hydraulic diameter.

Thank you to the National Science Foundation for supporting this work.

Great job making it to the end of the video and please reward yourself with a moment of zen.

I study fluid mechanics because I love water and healthy aquatic ecosystems. Whatever your passion is I hope it motivates you to continue your study of fluid mechanics