Hi and welcome to Engineering Earth. The topic of this video is the application of 1D continuity to the steady incompressible free surface flows.

What does this even mean? So let's just take a moment to unpack this topic, you may be surprised about how much you already know about it. What do I mean when I say 1D continuity? So when I say continuity of course I'm referring to application of the conservation of mass equations. And 1D flow means that I've got a velocity in one direction only, so by default this will become the streamwise component of velocity, u. So let's say that I have a finite control volume and I apply conservation of mass with one flow direction only, this means that I've got mass flowing in and out through the control volume in one direction only like I've shown here from left to right.

So when I talk about steady flow this means that my flow rate is not changing through the control volume, and therefore any mass that flows in also flows out at an equal rate. So m in is equal to m out and therefore storage in the control volume is zero.

So what that means is that this whole storage term, the first term in my conservation of mass equation, right here change of mass in my control volume over time, this is gone, it disappears because there can't be any storage when mass coming in is equal to mass going out. So I'm left only with the flux terms, the m dot is equal to of course a density, a velocity, and an area. So if I just think about this dimensionally, right, if I've got a density, multiplied by a velocity, multiplied by an area. And I replace each of these terms with their dimensions. My density is going to be a mass per cubic length, my velocity is a length per unit time, and my area is a length to the power of two. So overall I'm going to wind up with a dimensionality of a mass per unit time which is, of course, the dimensionality of my mass flux rate m dot. And so another way that I can just write this net mass flux out of my control volume, right, is I can since it's net mass flux out it's going to be my m dot leaving the control volume, m out, minus my m dot coming into the control volume is equal to zero. Little quick rearrangement here and I've got m out is equal to m in.

So now let's think about what this term incompressible means. Incompressible flow means that the density of my flow is going to be constant over both space and time and we know that mass flux, that term that we've just defined, m dot is going to be related to volumetric flow by the density of the fluid so we've used this equation many times already in this class.

So when I apply 1D continuity to a steady incompressible flow through my control volume we've already discussed that this is going to be m dot in equals m dot out. And if I think about what is, how can I express my mass flux coming in. Well it's going to be the density of my fluid coming into the control volume multiplied by the volumetric flow rate coming into the control volume and similarly my mass flux out is going to be the density of fluid leaving the control volume multiplied by my volumetric flow out. Now if we're working in an incompressible flow we know something about the density which is that it's not changing in either space or time. So I know that my density in has to be equal to density out um and so if I rewrite my conservation equation oops uh rather than writing in terms of mass flux if I just write it in terms sorry. If I just write it by replacing my mass flux with my density and volumetric terms and then I apply this information that I know, that my density is going to be the same in and out I am just left with volumetric flow coming in being equal to volumetric flow going out. So it's still true that mass flux in equals mass flux out but for an incompressible flow, like a liquid, I can also now rewrite my 1D continuity to a steady and incompressible flow as Q in equals Q out.

So by the way, what we have just done for this example of a finite control volume will also apply generally. So remember when we derived the differential form of the conservation of mass equation, the continuity equation, we arrived at a general expression that governs all fluid flows that looked something like this. Alright, so here's my general expression. And then when we wanted to apply this to a steady flow, of course all of the time dependent terms disappeared so we lost our first term here of our change in density over unit time just leaving us with the second term del rho v. So now if I want to apply my general form to incompressible flows, now I've got all terms that have a change in density over either space or time that are going to go away and I'm just left with del v is equal to zero.

Final thing that we want to discuss is how do we apply all of that to a free surface flow. Free surface flow means that the flow has a gas liquid interface. So let's imagine that I have got water that is flowing in a river. So this is my riverbed and up here this is the water surface so I'm looking at the river kind of from the side so that over here is the upstream and over here is the downstream. And my flow direction is going from upstream to downstream. So this is my streamwise component of velocity, u. And of course my water surface is a free surface. So I'll draw my little engineers boat denoting the free surface. Um now let's say that I want to apply that same finite control volume analysis to water that's flowing in my river and so I designate a control volume in a section of my river, like this. I know that the flow through my control volume is steady. So as long as there are no additional sources of water coming into my control volume, like a tributary entering or water withdrawing from a point within the control volume, then I know that the flow that's entering from upstream has to be equal to the flow exiting from downstream. So my application of the 1D continuity equation for a steady flow through a control volume of an incompressible fluid applies, and I've just got Q in is equal to Q out. How do I know that Q in is Q out? Well there can't be any storage happening in the control volume and there's just nowhere else for the mass to go it can't just disappear and more mass isn't being added from anywhere. So Q in has to be equal to Q out. So it's as simple as that, this long-term application of 1D continuity to steady incompressible free surface flows just boils down to Q in equals Q out. But we can even take it a little bit further from here because we also know that the volumetric flow rate, Q, is equal to the product of the cross-sectional area that the flow is moving through multiplied by the average speed of the flow through the cross-section. So if I turn my um my control volume and I look at either end, either the upstream end or the downstream end, let's say that at the upstream end, my cross-section looks something like this. I've got a wide cross-section of my river and let's say that, so this is my area in, and let's say that my area in is pretty big. And then let's go to my downstream cross-section. And my downstream cross-section is comparatively small. So by combining my 1D continuity to steady, incompressible, free surface flows with uh the knowledge that my volumetric flow rate is the product of the cross-sectional area and the average flow speed I can rewrite my conservation of mass equation to replace Q with AV. So I've got my cross-sectional area in multiplied by my average flow speed coming into the control volume and it has to be equal to my downstream area and V. Since I know that the cross-sectional area coming in is bigger than the cross-sectional area going out I can then make a conclusion also about the comparison of the flow speed going in versus out. What do you think is going to happen? Is the flow speed coming in going to be bigger than the flow speed going out or vice versa? Well, we know that the flow speed coming into my control volume has got to be smaller than the velocity going out because of the change in the cross-sectional area and the need to maintain conservation of mass as Q in equals Q out.

So thank you to the National Science Foundation for supporting this work.

Great job to you for reaching the end of this video.

Please reward yourself with a moment of zen. I study fluid mechanics because I love water and healthy aquatic ecosystems.Whatever your passion is, I hope it motivates you to continue your study of fluid mechanics