Hi and welcome to Engineering Earth. In this video we're going to review the linear momentum equation that we applied to control volumes in preparation for the differential analysis of momentum that we're going to undertake next.

Here's the general form of the linear momentum equation as applied to a control volume. So we can notice that there are some similarities to other conservation laws that we have applied, like to mass and energy. For instance, on the right hand side of this equation we have two terms. Um the first one, so this one this is our time rate change of linear momentum in the control volume. So another way of saying this is that it's the change of linear momentum in our control volume over some time period. And then our other term over here, this is our net flux of linear momentum out of the control volume over that same time. So another way of saying this is that it's like flux out minus flux in. So as mass is fluxing in and out of the control volume, moving at some speed and carrying momentum over some time we can calculate that net flux out and both of these terms are going to have units of momentum flux, right, so up here this is momentum flux. So a mass flux rate multiplied by a velocity and then this here this term is also momentum flux. Now here's the big difference, is that the sum of these two terms is the sum of external forces so on my left hand side over here I've got my sum of external forces. And that's quite different from most of the other conservation laws that we've studied, in which the sum of the two terms on the right hand side was always zero, meaning that there always had to be a balance between a net flux and a change in the control volume. So for linear momentum the momentum flux can either balance the change of momentum in the control volume or momentum can be unbalanced and the net force on the left hand side can balance them so there are a lot of interesting applications when momentum is balanced by force. You now may be wondering how can a sum of momentum flux be equal to a force? Well let's look at the dimensions. Um mass flux, let me give us a little bit of space here to write. Okay so a mass flux is going to have this m dot term which is going to have dimensions of a mass per unit time and my velocity, which is going to have dimensions of a length per unit time. So my mass flux dimensionality might look something like a mass time a length per unit time squared. So uh, for instance my mass flux units may be something like a kilogram per second, my velocity could be something like a meter per second, giving me a kilogram times a meter per second squared. And of course this dimensionality of a mass time a length per unit time is also the dimensions of force. So, for example we're used to thinking about force in terms of newtons that would be a kilogram times a meter per second squared, a mass time a length per unit time squared. Okay one final thing that I want you to notice in our general um momentum equation is that this V that shows up in my momentum flux term has that little r on it um indicating that this is the relative velocity of the fluid moving relative to the control volume so if our control volume is moving we may have to do that extra step of calculating the relative velocity to use in our momentum calculations. Now, let's think about what happens if we apply this momentum equation to a steady flow. So in a steady flow anything that's changing due to time is going to become zero, so I'm going to lose this entire first term.

And my momentum equation is now simplified and looks like this. So, let's say that I have a control volume okay here's my control volume, and this control volume is stationary, it's not moving. And I have a steady flow of fluid moving through it, but only some areas of the control surface are able to permit mass and momentum flux across the control surface. So, let's say that I have a couple of places where mass and momentum can flux in. So here I've got an inlet and I've got my momentum fluxing in through this inlet and maybe I've got another inlet over here, another place where momentum can come in. And then let's say I've got a couple of outlets, and I'll make my momentum out a different color, so in these locations I've got momentum that's fluxing out from the control volume and going outside. So remember that the right hand side of our equation in a steady flow is only this momentum flux. So flux out minus momentum flux in. And so now that I have this situation where flux can only happen in certain places I really just I don't need this integral anymore it's more information than I really need so I can rewrite the term on the right hand side by just adding up all of the places where momentum is fluxing out and then subtracting the sum of all of the places where momentum is fluxing in like this.

And now our momentum equation looks like this, I'm just rewriting what I wrote on the last slide. So I no longer have to integrate i just add up all of the momentum fluxes and so I'm going to just zoom in quickly on one of those inlets that was bringing mass into my uh control volume or maybe taking mass out. So if I zoom in on one of those little pipes right now and I'm looking at it in cross-section um I know that because of boundary layer physics and the no slip condition that my velocity distribution along the walls of this pipe is going to take this characteristic shape, that you guys have seen many times at this point. Where I've got really slow flows close to the wall of my pipe and then the magnitude of the velocity is increasing as we move closer and closer to the pipe center, as far away from those solid pipe walls as we can get. And I've got this nice parabolic, very nonuniform velocity profile that has developed in every one of those inlets that's bringing mass into or out of my control volume. Um and so I want to bring your attention to the velocity term in our momentum equation. So typically we're going to be using a mean velocity to calculate momentum, so I'm just going to add that here make it very clear that this is a mean velocity, and if I take a mean across a non-uniform velocity profile like this I'm always going to wind up with an underestimation of the true velocity right. So this V average is always going to be smaller than the actual velocity so if I use that V average to calculate momentum flux I'm going to systematically underrepresent the amount of momentum flux that's happening through each of my inlets and outlets. So the final thing that I'm going to do is I'm going to add a correction factor into my equation. On both my inlet and my outlets I'm going to put my beta parameter my momentum flux correction factor. And you guys may remember that we introduced a similar parameter, the alpha, the kinetic correction factor and we applied alpha to correct for the same problem the non-uniformity in our velocity profiles and the systematic underestimation of kinetic head. So um both beta and alpha are going to have a value of one when the velocity profile is uniform but the similarity ends there. Um they are mathematically different, they're calculated in different ways and they have different values. So for instance this um momentum flux correction factor when the flow is fully developed in laminar it's going to max out at about 1.3 and if you're working with a fully turbulent flow I suggest using a value for beta around 1.1 to 1.04, sorry that was 1.01 to 1.04. Whereas the kinetic correction factor can be as high as two in fully developed laminar flows and we've been using a value of about 1.05 for fully developed turbulent flows

Thank you very much to the National Science Foundation for supporting this work.

Great job making it to the end of the video.

And please reward yourself with a moment of zen. I study fluid mechanics because I love water and healthy aquatic ecosystems. Whatever your passion is, I hope it motivates you to continue your study of fluid mechanics