Hey everyone and welcome to Engineering Earth. In this video we're going to discuss common dimensionless parameters that are used in fluid mechanics.

One of the first things that we discussed in this class was related to the seven fundamental or primary dimensions, which are shown here. All other dimensions are made up of combinations of these fundamental dimensions. For example if we want to express velocity, that would be a length per unit time. And units are what we use to describe dimensionality. So we could use centimeters per second to describe velocity, if we're using the SI system of units. If we're using the US system of units maybe we would use something like an inch per second. Our discussions of dimensionality up to now have been mainly about dimensional homogeneity. So for instance, about how we can't combine unlike terms or about our need to ensure dimensionality is consistent from one side of the equation to the other. So let's expand upon this by expressing some common parameters that we use in fluid mechanics in terms of their primary dimensions.

Let's consider a parameter that we see often in fluid mechanics, shear stress, tau. What are the primary dimensions of shear stress? Well we know that it's a stress. So, therefore, it has to be a force per unit area, that's the definition of a stress. And then I further know that a force is equal to a mass time an acceleration. And so my primary dimensions of mass of course are mass, my primary dimensions of acceleration are going to be a length per unit time squared, right, so that could be something like a meter per second squared. Uh let's look at my primary dimensionalities of the second parameter, area. So this is going to be a length squared. So if I want to express my primary dimensionality of shear stress tau; and when I'm writing these little brackets around tau what I mean here, it's notation that means primary dimensionality of tau. So now I put together my force and my area. So I've got a meter times a length in my numerator and then I've got a time square and another length square in my denominator. I can simplify this a little bit by canceling out my length squared. And then I'm going to make one more slight modification move everything onto one line. This is just going to help me later on if I try to do things like dimensional analysis or something like that. And so here my primary dimensionality of shear stress is a mass to the power of 1, a time to the power of negative 2, and a length to the power of negative 1.

Let's look at now at an equation that we're all familiar with from our last unit. Here's the linear momentum equation for a fixed control volume within a finite number of inlets and outlets. So you remember that this equation equates the sum of external forces, over here, with the net outflux of momentum, so momentum leaving the control volume minus any momentum that's coming into the control volume through those inlets and outlets. So let's use primary dimensions to ensure that these terms are all indeed dimensionally consistent. I know I can probably do the right hand side subtraction because all of the variables are the same, right. I've got a beta, an m dot, and a velocity in each term so I can tell they're going to be dimensionally consistent. But is this dimensionally consistent to force on the left hand side of the equation? Let's find out by analyzing their primary dimensions. So I'll start with force, and we've already discussed that the primary dimensionality of force is going to be a mass times a length per time squared. So now I need to equate this with the dimensionality of my momentum terms. So my first term in the momentum sigma this isn't, doesn't have any dimensionality associated with it. It just means that I might have multiple momentum terms um leaving the control volume that I have to add up. So we don't have to worry about the sigma. The beta, this is just a coefficient, it has no dimensionality with it. But then we get to the m dot, m dot is a mass flux rate so we've got a mass to the power of one in the numerator and a time to the power of one in the denominator. And then I'm being, I'm multiplying this by velocity, so I've got a length to the power of 1 per unit time to the power of 1. And I get the same thing for my second momentum term, right. So overall I'm going to have a mass time a length per time squared. And so I can see that this is dimensionally consistent with my force term. Right I've got a mass to the power of one, a length to the power of one, a time to the power of negative 2 on the left hand side and on the right hand side I have the same thing. So indeed by looking at the primary dimensionality, I didn't put any units and any numbers into my equation, but I can still check and make sure that it's dimensionally consistent.

So now what about these dimensionless parameters? So the primary dimensionality of a dimensionless parameter is one. Or another way that I can say this is that any of those primary dimensions: mass, length, time, temperature, and so forth are all going to have an exponent of zero in order for my dimensionless parameter to be dimensionless, have a dimensionality of one. So let's look at an example of a dimensionless parameter that we have studied in this class. The Darcy friction factor, f, which we have defined as eight times the wall shear stress divided by the density of the fluid and the square of the velocity. So if I want to know my primary dimensionality of the Darcy friction factor my constant 8 has no dimensionality to it. We have already just discussed that the dimensionality of a shear stress is going to be a mass to the power of one, a length to the power of negative 1, and a time to the power of negative 2. And then on my denominator I'm going to have dimensionality of a rho of density So this is going to be a mass to the power of 1 multiplied by a length to the power of negative 3. Why is this? Well because density is a mass per unit volume. Mass has dimensionality of mass to the power of 1, volume has a dimensionality of a length to the power of negative 3. And then I've got my square of velocity. So of course I'm going to have a length squared and I'm going to have a time squared like this. And so let's uh do some cancelling out here, right, I've got a mass to the power of one here and a mass to the power of one here, I've got a length to the power of negative 3 plus a length to the power of 2 that are going to cancel with my length to the power of negative 1 in the numerator. And then finally my 2 times to the power of negative 2 are going to cancel one another out. Leaving me with a primary dimensionality of my Darcy friction factor of mass to the power of zero, length to the power of 0, time to the power of 0, or dimensionality 1. Proving that my Darcy friction factor is indeed dimensionless.

Another dimensionless parameter that we have analyzed in this class is the famous Reynolds number, that ratio of inertial to viscous forces that we use to characterize flow regimes. We've got density, velocity, and a length scale in the numerator and then viscosity on the denominator. If I want to know my primary dimensionality of the Reynolds number, again I'm going to have my mass to the power of 1 length to the power of negative 3 up here for density. My velocity again is going to be length to the power of 1, time the power of negative 1. And my length scale this is just length to the power of 1. In my denominator I've got viscosity, mass to the power of 1 length to the power of 1 time to the power of negative 1. I'll do a little bit of simplifying. So in my numerator I'm left with a mass to the power of 1, a length 1, and a time negative 1. And I have the same thing in my denominator. And so my primary dimensionality of my Reynolds number once again is going to be mass to the power of zero, length to the power of zero, time to the power of zero. It is indeed dimensionless.

This is so much fun, so why stop? Let's look at one more dimensionless parameter that we've discussed. The drag coefficient, that is appearing in our quadratic drag law that is relating drag force to one half the drag coefficient the density of the fluid, the square of the velocity, and the cross-sectional area. So I can just rearrange this to solve for my dimensionless parameter, the uh the drag coefficient, and now I can start looking at primary dimensionality of my drag coefficient. So again the constant here, 2, has no dimensionality associated with it. I have the dimension of force, which we already discussed is a mass to the power of one, length to the power of 1, time to the power of negative 2. In my denominator I've got dimensionality of uh density mass to the power of 1, length to the power of negative 3. I've got a dimensionality of a square of velocity, so a length to the negative, or length to the power of two, um time to the power of negative 2.

and then finally I've got dimensionality of that area which is a length squared. A little bit of algebra simplification, and then once again I can see that the primary dimensionality of my drag coefficient is: mass to the power of zero, length to the power of zero, and time to the power of zero.

Thank you to the National Science Foundation for supporting this work.

Great job, you, making it to the end. Please reward yourself with a moment of zen. I study fluid mechanics because I love water and healthy aquatic ecosystems. Whatever your passion is, I hope it motivates you to continue your study of fluid mechanics