Hi everyone! This video covers a review of the Taylor series expansion, which is something that you've learned about previously in your Calculus classes.

So, I'm going to write the definition of a Taylor series. Let's say that we know the value of some function, f, at some location, x, like this and we want to predict an unknown value of that same function f at some location x plus a. So, we can apply a Taylor series expansion to get us closer to the true unknown solution of f of x plus a. So, I'm adding my Taylor series expansion to f of x and what you can see is it's an increasingly smaller series of terms; each one that marches us closer and closer to the true solution of f of x plus a. And I'm going to stop after the fourth term here um, but this keeps going and going, it's an infinite series. So, as you can see, the second term is multiplied by a, right so this is our distance away from x. And then our third term is multiplied by that same distance a squared and then divided by 2 factorial. My fourth term is multiplied by a to the power of 3 and then divided by a 3 factorial. So, if a is a small value, if we choose a small value away from x that we want to predict, each term is going to become smaller and smaller and smaller. So, the Taylor series um has applications in numerical modeling. So, for instance let's say that you are running a model uh that requires solution of some sort of complex function, like a nonlinear equation or a partial or ordinary differential equation um and your true solution is challenging, uh you want to predict the value of an unknown location. The Taylor series can move that return solution closer and closer to the true value and minimizing error that would tend to drift your model prediction further and further from the true value. So, let's say you're running your model over a time domain or a spatial domain um that drift in the model due to the model error can wind up uh giving you a prediction that's pretty far away from the true value. Taylor series expansion is a good numerical solution to keep um your model prediction as close to the true value as possible.

So, the rules of Taylor series expansion: um the function f and all of its derivatives that we're applying have to exist, that's pretty uh self-explanatory, um and they have to be continuous in the domain that we're applying it, so in the domain of x or x plus a. And I also want to point out that the Taylor series does not require that we know the function itself, we only have to know its value, for instance the value of function um at point x is our first term, you know. This second term, this is the value of the first derivative of f of, of the function f at point x. And then, our third term this is the value of the second derivative of function f at point x. So, nowhere in here do we actually need to know uh the value itself, or the function itself.

So, let's do an example. So, in our example let's say that we know the value of function f when x is equal to three and its value is 100, and we know the value of the first derivative of the function f at x is equal to 3 and that value is 75, and we know the value of the second derivative of function f at x equals 3 and it is 50, and then the value of the third derivative of function f at x equals 3 is 25,and let's just say that we know that the value of any higher order derivatives of function f at uh x equals 3 are all zero. So there's no more terms that we're going to need to apply in our Taylor series expansion. And now let's say we want to predict the value of function f at 4, this is our unknown. So, we are going to apply a Taylor series expansion to estimate the value of function f when x is equal to 4. So, in our problem x is equal to 3, a is equal to 1, so the value of x + a is 3 + 1 equals 4.So if I want to know the value of f of x + a that would be my unknown, right, my value of function f when x is equal to 4.So I'm just going to substitute these variables x is 3, a is 1, into the Taylor series expansion. So my first term is going to be f of x, so my f of three term, my second term is the first derivative of f of 3 multiplied by a, which has a value of 1. My second term will be the second derivative of X multiplied by a, this time squared, and my fourth term will be the third derivative of function f at x = 3, oops at 1 cubed, over 3 factorial. And so now just expanding these out, substituting, I know that my value of f of 3 is 100 my value of f prime of 3 is 75 times 1 divided by 1, plus my f double prime of 3 which was 50 and then my third derivative of f my value is 25. And so, each of my terms becomes smaller and smaller and smaller. I barely have room to write it down here, my first term is 100, my second term is 75, my third term is 25, my fourth term is going to be 25 over 6 altogether I'm going to get an f, a value of f at x equals 4 equal to 204 point 1667. So, I've got decreasing value of each term in the expansion becomes smaller and smaller and smaller each one gets me closer and closer and closer to the true value of my function when x is equal to 4.

So let's do a quick just sneak peek it at how we can apply the Taylor series expansion to fluid mechanics. And so, in this example let's say that we have got a flow that is moving through this domain shown here, I've just it as a six-sided cube. And each side of my cube has a length that's an infinitesimal length in the space X, Y, and Z. Oops, I see a mistake right here, this should be dx not dz. Right, so my length is dx, dy, dz. Um and I've got a fluid that is coming into my cube domain through the left face side and it's flowing out through the right face. And so, I want to think about the mass flux that's coming into my flow domain and then going out here, through the right, and if I think about my mass flux right my term m dot. I know that the dimensions of m dot are going to be a mass per unit time. And so, for instance, I could represent a mass per unit time that's going through my left face and through the right face of this cube as being equal to the density, the velocity in x, u, and the area of the cube that it's fluxing through, in this case it's going to be dydz. And this is going to tell me about mass flux coming in through the left face. And I want to point out that this rho u right here is specific to on this left face and then over here on the other side the mass going out it's going to be rho u on the right face, multiplied by the area which again is going to be dydz. Okay, so now my trouble becomes estimating what is that mass flux on the right and left face. Uh before I go on to that let's back up, just I think it can be confusing sometimes to understand um how come this rho u dydx gives me a mass flux. So if I think about this as a density times a velocity multiplied by a little differential area da, represented by dydz, um and I think about the units, or rather the dimensions, right my density this would be something like a mass per cubic length my velocity this would be a mass, I'm sorry not a mass, this would be a length per unit time and my da this would be a length to the power of two leaving me with a mass per unit time, which of course is my mass flux. So, let's say that I know the value of the density of the fluid and the velocity vector at this point, I'll call it point x, right in the middle of my cube volume. And let's say that right here I've got my velocity in u in v and in w and I know at point x that my density is rho and velocity is u, v, w. Now, the trouble is that I may know the value of density and velocity here at the center of the cube but if I move just over a little bit from that center over to the right hand face, now I don't really know exactly what the density and the velocity is. I haven't gone far, right I've just gone this little distance like half of dx right, from where my point x is to my right-hand face that's only a distance of dx over 2. But I still don't know exactly what density and velocity are, so how can I estimate what the mass flux is. Well, here is where I can bring in my Taylor series expansion to help me. Right, so if I want to know the value of a function at some value x, which is going to be my point x, and I'm moving some distance over a, and we've already discussed that that distance that I'm moving a is going to be dx over 2, oops I got to fix this mistake again this is supposed to be a dx, right so my distance is dx over 2. So, I'm writing f of x plus dx over 2, that's going to give me the value of my function over on the right-hand face of my cube. And so, it's going to be whatever that value was at f plus my first derivative of the function at x multiplied by my distance a, and so on so on; the Taylor series expansion that we've already gone over. So, what is it that I want to know? I want to know the combination of density and velocity on the right-hand face of my cube. And this is going to be my f of x plus dx over 2, and I'm going to estimate this as my value at x rho u plus the first derivative of rho u multiplied by my distance which is dx over 2. And then if I want to expand to a second ordered term and do my second derivative, this is going to be multiplied of course by dx over 2 to the power of 2, so each one of my terms is going to be getting you know smaller and smaller as I go along. And remember that, you know, we're dealing with this infinitesimal volume of sides dx, dy, dz which are already very short, and then we're moving over a distance of dx over 2 which makes it even smaller. And so once I get out to my third term in the Taylor series expansion and I'm squaring dx over 2, then it becomes tiny, so it's not uncommon for us to be able to approximate our true solution using just the first two terms of the Taylor order uh the Taylor expansion because all of the other higher order terms become so small so quickly.

So, thank you to the National Science Foundation for supporting this work under uh Grant 2335802.

Great job reaching the end of this video, and please reward yourself with a moment of Zen. I study fluid mechanics because I love water and healthy aquatic ecosystems. Whatever your passion is, I hope it motivates you to continue your study of fluid mechanics. Thank you.