Hi, and welcome to Engineering Earth. This video reviews the Reynolds Transport Theorem, which we recently derived together in class based on analysis of flows through a closed system approach and an open system control volume.

In class, in chapter 4, we talked about the two different approaches that we can take to analyze moving fluid, the first being the Lagrangian approach, where our analysis follows a control mass. So, let's say I've got a control mass, right here in a closed system and this control mass is moving with respect to time. Since I'm taking this Lagrangian approach, I have to keep track of the position vector and the velocity vector of our control mass as these things change over time. Right, um and this is sometimes called a systems approach to fluid analysis because it treats that control mass as a closed system. And this Lagrangian systems approach can get kind of complicated when matter is in liquid or a gaseous state, because every particle of this fluid is able to move independently from its neighbors. So, it's not like when we throw something solid, like a football. And then the movement of every particle of that football can be described by the same velocity vector, kind of like what I've shown here. If that whole control mass moves in one trajectory I can use one position vector, and one velocity vector that changes over time, and it will describe the change in position of that entire control mass. But, when we're working in a fluid then we might need a different velocity vector or position vector for every particle of this control mass. So, you can see how this gets complicated pretty quickly. But fortunately, we have a second option that we can use to analyze moving fluids and this is called the Eulerian approach. So, in our or Eulerian approach what we're going to do is instead of just following a system of mass, we are going to designate a control volume, abbreviated by CV. And remember our control volume is a region of space and everything inside of the control volume is separated from everything in the surroundings outside by this control surface. And mass, or energy, or momentum can get into our control volume by flowing across that control surface and going in and then it can flux out, like that. And so, this Eulerian approach tends to be a more practical approach to some of the most common problems that we have in fluid mechanics.

But, most of the equations of motion that you may have studied in your prior courses were written kind of more for a Lagrangian description of control mass. So, to translate these known equations into a usable form for a control volume analysis, in class we derived the Reynolds Transport Theorem, that's shown here. And since we've already spent a good amount of time talking about the Reynolds Transport Theorem during the derivation in class this video is just going to emphasize a few of the key points that people find confusing about uh the Reynolds Transport Theorem. So, the first thing that sometimes confuses people right off is you know: What is this big B here? What is this little b over here? What are these variables? We see a lot of familiar variables in the equation right we know t for time, we know rho for density. But this big b, what is this? So this is a variable that can be any extensive property for a system of mass. And you'll remember, that at the beginning of this course we defined extensive properties as those that depend on the extent of the system, or the size of the system. So mass, energy, and momentum are all extensive properties. So if you had um a system of mass and then you split that system into two uh systems of mass, each with one half of the mass of the original, then clearly the mass, the energy, and the momentum of those two systems is going to be different than the mass of the original system. So that's what makes these um you know mass, energy, and momentum extensive properties, because they depend on the extent of the system. So then, little b is the corresponding intensive property that goes with big B. Right so, if we define big B as the extensive property then the corresponding intensive property is going to be uh the same property but now it doesn't depend on the size of the system any longer. Right, so we can think about if we take big B: mass, energy, or momentum, and we divide that per unit mass that's going to be the definition of the corresponding intensive property, little b.

Okay, so back looking at our Reynolds transport theorem, um you're going to notice that there are three terms in our equation. So one term on the left hand side, and this is the time rate change of that extensive property, B, of a system of mass. So if we're applying the Reynolds transport theorem to analyze mass, energy, or momentum, well these are all conservative quantities, right, so the change um of mass, or energy, or momentum over time within a closed system always is going to be zero. Okay, and so then on the right hand side we've got two terms. The first term, this is the change in mass, energy, or momentum in a control volume over time. So, we've got this little b, intensive property, it can be it can be mass, it can be energy it can be momentum. And this is in our control volume over time. So you can think of this um as the storage of mass or energy or momentum inside the control volume. So, remember the change of water level that we drew in our bathtub control volume? That's what this term is describing. So, when the tub is filling up, the water level is going up over time and in a case like that, this term this uh this second term in our equation right here would be positive. Um, so when the tub was draining the water level was going down over time, in that case then the second term would be negative. So now let's look at the last term: second term on the right hand side of the equation. Um, and so this is the net flux of that intensive property, b, so mass, energy, or momentum per unit mass out of the control volume. How does mass, energy, or momentum flux out of the control volume? Well, it has to cross over that control surface. So, remember the faucet and the drain in our bathtub control volume. The water leaving through the drain, this is mass that is fluxing out. Water coming in through the faucet that would be mass that's fluxing in. So if we subtract the mass that's coming in from the mass that's draining, that would be this third term the net outflux of mass.

Now let's take a look up here at this term. This always causes some questions. First of all, what is this little r on our velocity? The r means that we are concerned with the velocity of the fluid uh moving relative to the control surface. So, remember when we talked about the control surface we defined that it can be really flexible it can be moving it can be deforming. So that means that the velocity of matter that's crossing over the control surface could potentially be moving at a speed that's different from the speed of the fluid relative to a fixed reference point, like on the ground. So if the control volume is not moving, if it's stationary, then the velocity of the fluid relative to the control surface uh is the same as the velocity relative to a fixed reference point. But this little r here is just to remind us that if our control surface happens to be moving, then we want to use the velocity of the fluid relative to the control surface. So that's all that means. Um, so now let's think about like this dot product right. So we've got this little n right here. What is n? So n this is our unit normal vector and so it has a magnitude of one and a directionality that is always oriented 90 degrees from the control surface and directed outwards. So, let's say we have a control volume with a control surface that looks something like this.Â And all of our, oops sorry let me label this, this is my control volume and my control volume is going to have unit normal vectors directed outward from each of its boundaries at an angle that's normal to the boundary on all sides, each one has a magnitude of one. And so V dot n is the dot product of the velocity vector and that unit normal vector. So we've got the magnitude of velocity, the magnitude of our unit normal vector, which of course is just going to be one, multiplied by the cosine, sorry the cosine of the angle theta between the velocity vector and the unit normal vector. So why is it V dot n, and not just V? Well, again if we look at this third term of the Reynolds transport theorem here, um remember that we defined this as the net flux out of the control volume. So, the net flux of mass, energy, or momentum that's going out of the control volume. Um so that's going to be the outflows minus the inflows. So in order for mass, energy, or momentum to enter or exit into our control volume this requires that mass has to cross over the control surf surface and only the component of a velocity vector that is normal to the control surface is going to contribute to mass flux over that control surface. So the V is our velocity vector, let's say we've got a flow that's coming in at an angle, like this, and it's hitting the boundary of our control volume like that. So I could decompose this velocity vector into its components that are normal and tangential to the surface right, so this one is going to be our normal component and this one over here is going to be the tangential component. So instead of the dot product of the normal vector this would be the dot product of the tangent vector, tau. So the magnitude of the dot product is going to tell us about how much the velocity vector is contributing to mass flux and therefore, energy and momentum flux over the control surface. And its directionality is going to tell us about whether that velocity is contributing to mass flux going into the control volume or mass flux that's coming out of the control volume. Um, so what do I mean like about this? Let me explain, let me get rid of this quickly. So let's say that we have a velocity um that's moving like this. And so the angle theta between my unit normal vector and my velocity vector in this case is going to be 90 degrees. So if my angle theta equals 90 degrees, then my cosine theta, oops sorry, my cosine theta is equal, not to one, but to zero. And so what that's going to tell me is that there is no flux across, you know, this boundary of my control volume right here because of this velocity vector. So all of the mass is going to be moving tangentially to this boundary and none of it will cross across that boundary and enter from outside to inside of the control volume. Okay, but then when theta is zero, so let's think about the components of that same flow that's moving like this. And then out here it's coming like that, so on my top boundary up here the angle between my unit normal vector and my velocity vector is going to be zero. And if theta is equal to 0 degrees, then my cosine of theta now it's equal to one, and this is going to denote an outflux of mass from the control volume. Right, see my velocity vector here is going out of the control volume. If it were coming in like down here now, see what happens, now my angle theta is equal to 180 degrees and my cosine theta is equal to negative 1. So, in this case my velocity vector and my unit normal vector are oriented at 180 degrees from each other. Cosine of 180 degrees is negative 1 and so that indicates that I've got an influx of mass into my control volume. So anytime that my angle theta is greater than 90 degrees, my cosine theta is going to be negative and this will indicate that I've got mass coming into the control volume. Anytime that my angle theta is less than 90 degrees, then cosine theta will be greater than zero, and this denotes mass leaving the control volume. So this is why we have the dot product of velocity, is to indicate, first of all the component of that velocity vector that's contributing to the mass flux across the control volume, across the control surface. But, it's also to keep track of the directionality right, because this third term in our Reynolds transport theorem it's specific about directionality. It's the amount of mass, energy, or momentum that's leaving the control volume. So mass, energy, momentum out minus mass, energy, or momentum coming in. So we have to really keep track of which direction that mass energy or momentum is flowing at any given time.

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And great job reaching the end of the video. Please reward yourself with a moment of Zen with this beautiful mountain river from the Luquillo LTER station in Puerto Rico. I study fluid mechanics because I love water and healthy aquatic ecosystems. Whatever your passion is, I hope it motivates you to continue your study of fluid mechanics.