

# Reynolds Transport Theorem

## Engineering Fluid Mechanics

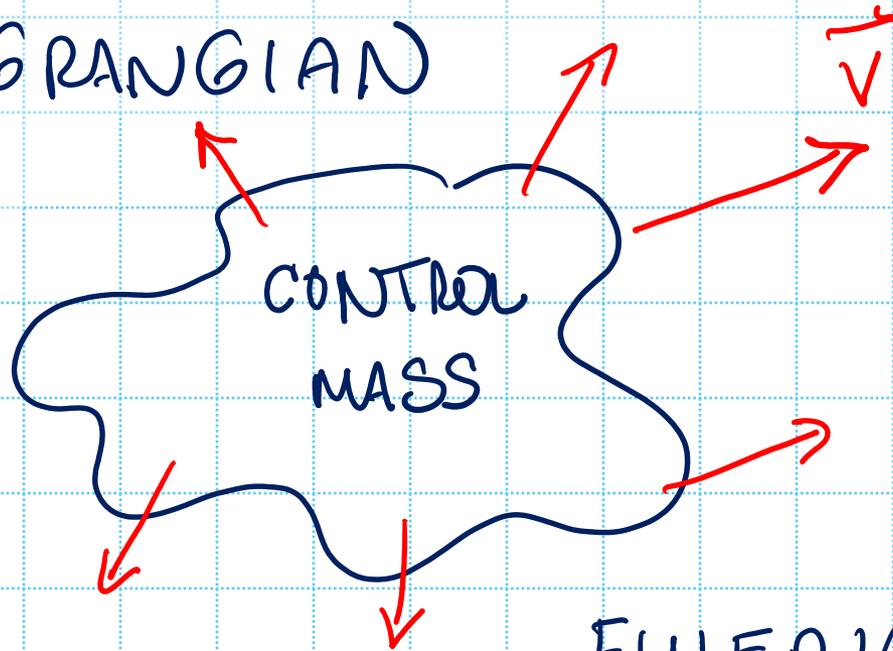
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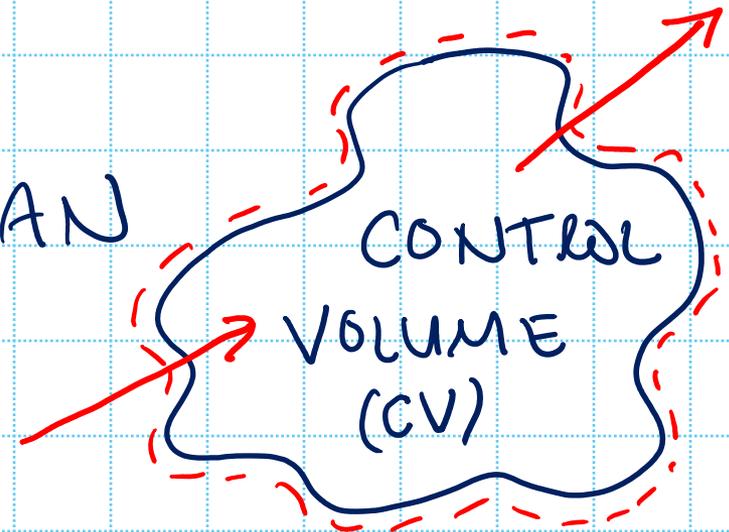
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LAGRANGIAN



EULERIAN



Lagrangian (systems) and Eulerian  
(control volume) analysis



$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho b dV + \int_{CS} \rho b (\vec{V}_r \cdot \hat{n}) dA$$

$B$  = ANY EXTENSIVE PROPERTY OF  
A SYSTEM OF MASS  
(MASS, ENERGY, MOMENTUM)

$b$  = THE CORRESPONDING INTENSIVE  
PROPERTY

$$\frac{B}{m} = b$$

Reynolds transport theorem



$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho b \, dV + \int_{CS} \rho b (\vec{V}_r \cdot \hat{n}) \, dA$$

CHANGE OF B OVER TIME IN A SYSTEM OF MASS (IF B IS MASS, ENERGY, MOMENTUM WILL BE 0) = CHANGE MASS IN CV OVER TIME ENERGY, MOMENTUM + NET FLUX OF (MASS ENERGY, MOMENTUM) out of CV

# Reynolds transport theorem

VELOCITY  
RELATIVE TO  
CS

$$\frac{dB_{sys}}{dt} = \frac{d}{dt} \int_{CV} \rho b \, dV + \int_{CS} \rho b (\vec{V}_r \cdot \hat{n}) \, dA$$

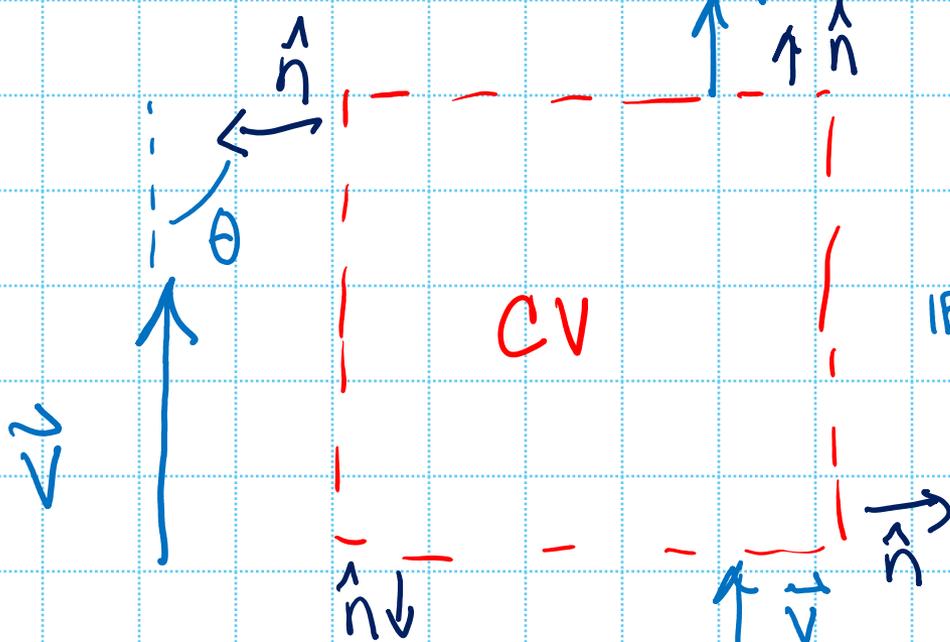
UNIT NORMAL  
VECTOR

$$|\vec{V}| |\hat{n}| \cos \theta$$

IF  $\theta = 90^\circ$   
 $\cos \theta = 0 \rightarrow$  NO FLUX

IF  $\theta = 0^\circ$   
 $\cos \theta = 1 \rightarrow$  OUTFLOW

IF  $\theta = 180^\circ$   
 $\cos \theta = -1 \rightarrow$  INFLOW



# Reynolds transport theorem



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# THE END



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